Mathematics-8

1 Rational Numbers

Exercise 1.1

1. (a) Given rational number $=\frac{-56}{104}$

HCF of 56 and 104 = 8

Dividing both numerator and denominator by 8, we get

$$-\frac{56\div 8}{104\div 8}=\frac{-7}{13}$$

Thus, $\frac{-56}{104}$ is expressed in its lowest terms as $\frac{-7}{13}$.

(b) Given rational number $=\frac{65}{-156}=\frac{-65}{156}$.

HCF of 65 and 156 = 13

Dividing both numerator and denominator by 13,

we get $$\frac{-65}{156}=-\frac{65\div 13}{156\div 13}=\frac{-5}{12}$$

Thus, $\frac{-65}{104}$ is expressed in its lowest terms as $\frac{-5}{12}$.

(c) Given rational number $=\frac{-84}{-91}=\frac{84}{91}$

HCF of 84 and 91 = 7.

Dividing both numerator and denominator by 7, we get

$$\frac{84}{91}=\frac{84\div 7}{91\div 7}=\frac{12}{13}$$

Thus, $\frac{84}{91}$ is expressed in its lowest terms as $\frac{12}{13}$.

(d) Given rational number $=\frac{65}{125}$

HCF of 65 and 125 = 5.

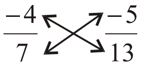
Dividing both numerator and denominator by 5, we get

$$\frac{65}{125}=\frac{65\div 5}{125\div 5}=\frac{13}{25}$$

Thus, $\frac{65}{25}$ is expressed in its lowest terms as $\frac{13}{25}$.

2. (a) Given numbers are $\frac{-4}{7}$ and $\frac{-5}{13}$.

Using cross multiplication, we get



$-4 \times 13 \qquad\qquad -5 \times 7$

$-52 \qquad\qquad\quad -35$

Since, $-52 < -35$

So, $\frac{-4}{7} < \frac{-5}{13}$

(b) Given numbers are -8 and $\frac{-15}{6}$.

Using cross multiplication method, we get

$$-\frac{8}{1} \times \frac{-15}{6}$$

$-8 \times 6 \qquad\qquad -15 \times 1$

$-48 \qquad\qquad\quad -15$

Since, $-48 < -15$

So, $-8 < \frac{-15}{16}$

(c) Given numbers are $\frac{-3}{8}$ and $\frac{6}{-11}$.

Using cross multiplication method, we get

$$\frac{-3}{8} \times \frac{6}{-11}$$

$-3 \times 11 \qquad\qquad -6 \times 8$

$-33 \qquad\qquad\quad -48$

Since, $-33 > -48$

So, $-\frac{3}{8} > \frac{-6}{11}$

(d) Given numbers are $\frac{-4}{13}$ and $\frac{9}{10}$.

Since, the positive rational number is always greater than negative rational number.

So, $\frac{-4}{13} < \frac{9}{10}$

3. (a) The given rational numbers are $\frac{-4}{7}, \frac{5}{-42}, \frac{-3}{28}$ and $\frac{-2}{21}$.

LCM of 7, 42, 28 and 21 $= 84$

Now, making the denominators of all rational numbers 84.

$$\frac{-4}{7} = \frac{-4 \times 12}{7 \times 12} = \frac{-48}{84}, \frac{5}{-42} = \frac{-5 \times 2}{42 \times 2} = \frac{-10}{84}$$

$$\frac{-3}{28} = \frac{-3 \times 3}{28 \times 3} = \frac{-9}{84} \text{ and } \frac{-2}{21} = \frac{-2 \times 4}{21 \times 4} = \frac{-8}{84}$$

Now, comparing the numerator of the above formed rational numbers, we get

$-48 < -10 < -9 < -8$

So, $\frac{-48}{84} < \frac{-10}{84} < \frac{-9}{84} < \frac{-8}{84}$

or $\frac{-4}{7} < \frac{-5}{42} < \frac{-3}{28} < \frac{-2}{21}$

Hence, $\frac{-4}{7} < \frac{-5}{42} < \frac{-3}{28} < \frac{-2}{21}$ is the required ascending order.

(b) The given rational numbers are $\frac{-1}{4}, \frac{5}{-12}, \frac{-7}{24}$ and $\frac{-9}{8}$.

LCM of $4, -12, 24$ and $8 = 24$
Now, making the denominators of all rational numbers 24.
$-\frac{1}{4} = \frac{-1\times 6}{4\times 6} = \frac{-6}{24}, \frac{-5}{12} = -\frac{5\times 2}{12\times 2} = -\frac{10}{24}$
$-\frac{7}{24} = \frac{-7\times 1}{24\times 1} = \frac{-7}{24}$ and $\frac{-9}{8} = -\frac{9\times 3}{8\times 3} = \frac{-27}{24}$
Now, comparing the numerator of the above formed rational numbers, we get

$$-27 < -10 < -7 < -6$$

So, $$\frac{-27}{24} < \frac{-10}{24} < \frac{-7}{24} < \frac{-6}{24}$$

or $$\frac{-9}{8} < \frac{-5}{12} < \frac{-7}{24} < -\frac{1}{4}$$

Hence, $\frac{-9}{8} < -\frac{5}{12} < -\frac{7}{24} < -\frac{1}{4}$ is the required ascending order.

(c) The given rational numbers are $\frac{-5}{6}, \frac{10}{3}, \frac{-7}{-3}$ and $\frac{13}{8}$.

LCM of $6, 3, 3$ and $8 = 24$
Now, making the denominators of all rational numbers 24.
$\frac{-5}{6} = \frac{-5\times 4}{6\times 4} = \frac{-20}{24}, \frac{10}{3} = \frac{10\times 8}{3\times 8} = \frac{80}{24}, \frac{-7}{-2} = \frac{7\times 12}{2\times 12} = \frac{84}{24}$ and $\frac{13}{8} = \frac{13\times 3}{8\times 3} = \frac{39}{24}$
Now, comparing the numerators of the above formed rational numbers, we get

$$-20 < 39 < 80 < 84$$

So, $$\frac{-20}{24} < \frac{39}{24} < \frac{80}{24} < \frac{84}{24}$$

or $$\frac{-5}{6} < \frac{13}{8} < \frac{10}{3} < \frac{-7}{-2}$$

Hence, $\frac{-5}{6} < \frac{13}{8} < \frac{10}{3} < \frac{-7}{-2}$ is the required ascending order.

4. . (a) The given rational numbers are $\frac{-7}{10}, \frac{23}{-5}, \frac{-2}{15}$ and $\frac{-11}{30}$.

LCM of $10, 5, 15$ and $30 = 30$
Now, making the denominators of all rational numbers 30.
$\frac{-7}{10} = \frac{-7\times 3}{10\times 3} = \frac{-21}{30}, -\frac{23}{5} = -\frac{23\times 6}{5\times 6} = \frac{-138}{30}, -\frac{2}{15} = \frac{-2\times 2}{15\times 2} = \frac{-4}{30}$ and
$\frac{-11}{30} = \frac{-11}{30}$
Now, comparing the numerator of the above-formed rational, we get

$$-4 > -11 > -21 > -138$$

So, $$-\frac{4}{30} > -\frac{11}{30} > \frac{-21}{30} > \frac{-23}{5}$$

or $$-\frac{2}{15} > -\frac{11}{30} > -\frac{7}{10} > \frac{-23}{5}$$

Hence, $\frac{-2}{15} > -\frac{11}{30} > -\frac{7}{10} > -\frac{23}{5}$ is the required descending order.

(b) The given rational numbers are $\frac{-11}{5}, \frac{13}{-8}, \frac{-7}{4}$ and $\frac{17}{-10}$.

LCM of 5, 8, 4 and 10 $=$ 40

Now, making the denominators of all rational numbers 40.

$\frac{-11}{5} = \frac{-11\times 8}{5\times 8} = \frac{-88}{40}$, $\frac{13}{-8} = \frac{-13\times 5}{8\times 5} = \frac{-65}{40}$, $\frac{-7}{4} = \frac{-7\times 10}{4\times 10} = \frac{-70}{40}$ and $-\frac{17}{10}$ $= \frac{-17\times 4}{10\times 4} = \frac{-68}{40}$

Now, comparing the numerator of the above-formed rational numbers, we get

$$-65 > -68 > -70 > -88$$

So, $$\frac{-65}{40} > \frac{-68}{40} > \frac{-70}{40} > \frac{-88}{40}$$

or, $$\frac{13}{-8} > \frac{-17}{10} > \frac{-7}{4} > \frac{-11}{5}$$

Hence, $\frac{-13}{8} > \frac{-17}{10} > \frac{-7}{4} > \frac{-11}{5}$ is the required descending order.

(c) The given rational numbers are $\frac{-5}{6}, \frac{13}{-18}, \frac{-7}{12}$ and $\frac{-15}{24}$.

LCM of 6, 18, 12 and 24 $=$ 72

Now, making the denominators of all rational numbers 72.

$\frac{-5}{6} = \frac{-5\times 12}{6\times 12} = \frac{-60}{72}$, $\frac{13}{-18} = \frac{-13\times 4}{18\times 4} = \frac{-52}{72}$, $\frac{-7}{12} = \frac{-7\times 6}{12\times 6} = \frac{-42}{72}$ and $\frac{-15}{24}$ $= \frac{-15\times 3}{24\times 3} = \frac{-45}{72}$

Now, comparing the numerators of the above-formed rational numbers, we get

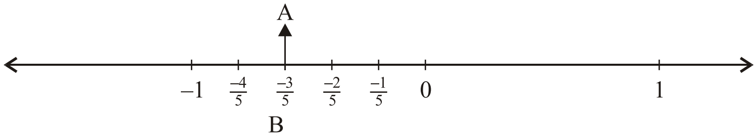
$$-42 > -45 > -52 > -60$$

So, $$\frac{-42}{72} > \frac{-45}{72} > \frac{-52}{72} > \frac{-60}{72}$$

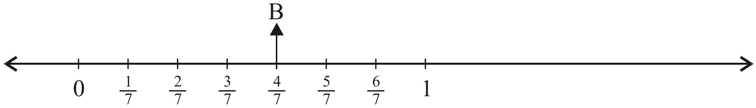
or $$\frac{-7}{12} > \frac{-15}{24} > \frac{-13}{18} > \frac{-5}{6}$$

Hence, $\frac{-7}{12} > \frac{-15}{24} > \frac{-13}{18} > \frac{-5}{6}$ is the required descending order.

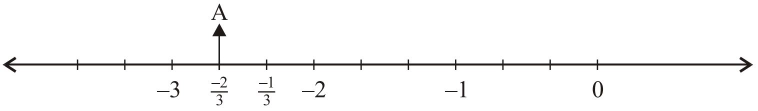
5. (a) $\frac{-3}{5}$



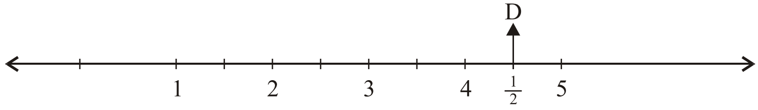
(b) $\frac{4}{7}$



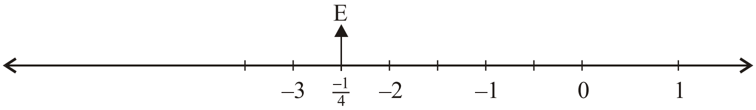
(c) $\frac{-8}{3} = -2\frac{2}{3}$



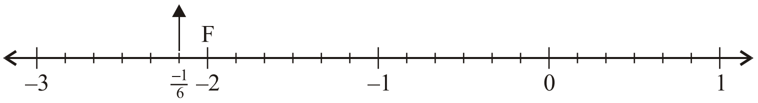
(d) $4\frac{1}{2}$



(e) $\frac{-9}{4} = -2\frac{1}{4}$



(f) $-2\frac{1}{6}$



6. (a) $\left|\frac{2}{3}\right| - \left|\frac{-7}{5}\right| = \frac{2}{3} - \frac{7}{5} = \frac{2\times 5 - 7\times 3}{15} = \frac{10-21}{15} = \frac{-11}{15}$

(b) $\left|\frac{8}{15}\right| + \left|\frac{-3}{16}\right| = \frac{8}{15} + \frac{3}{16} = \frac{8\times 16 + 3\times 15}{240} = \frac{128+45}{240} = \frac{173}{240}$

(c) $-|-(-2)| + \left|-\frac{1}{6}\right| = -|2| + \frac{1}{6} = -2 + \frac{1}{6} = \frac{-2\times 6 + 1}{6} = \frac{-12+1}{6} = \frac{-11}{6}$

7. (a) $\left|\frac{7}{5}\right| \square \left|\frac{-8}{5}\right|$

$\frac{7}{5} \quad \frac{8}{5}$ [By cross multiplication]

$7\times 5 \qquad 5\times 8$

$35 \qquad 40$

Since, $35 < 40$

So, $\left|\frac{7}{5}\right| < \left|\frac{-8}{5}\right|$

(b) $\left|\frac{-11}{4}\right| \square \left|\frac{-3}{16}\right|$

$\frac{11}{4} \quad \frac{3}{16}$ [By cross multiplication]

$11\times 16 \qquad 4\times 3$

$176 \qquad 12$

Since, $176 > 12$

So, $\left|\frac{-11}{4}\right| > \left|\frac{-3}{16}\right|$

(c) $\left|\frac{-8}{12}\right| \square \left|\frac{-10}{-15}\right|$

$$\frac{8}{12} \times \frac{10}{15}$$

$$\begin{array}{ll} 8\times 15 & 12\times 10 \\ 120 & 120 \end{array}$$

Since, $120 = 120$

So, $\left|\frac{-8}{12}\right| = \left|\frac{-10}{15}\right|$

(d) $\left|\frac{-5}{6}\right| \square \left|\frac{6}{-8}\right|$

$$\frac{5}{6} \times \frac{6}{8}$$

$$\begin{array}{ll} 5\times 8 & 6\times 6 \\ 40 & 36 \end{array}$$

Since, $40 > 36$

So, $\left|\frac{-5}{6}\right| > \left|\frac{6}{-8}\right|$

8. $|x| = |-x|$

(a) $x = \frac{2}{3}$ (Given)

LHS $= |x| = \left|\frac{2}{3}\right|$ RHS $= |-x|$

$= \frac{2}{3}$ $= \left|-\frac{2}{3}\right| = \frac{2}{3}$

Hence, LHS $=$ RHS

(b) $x = \frac{-3}{4}$

LHS $= |x| = \left|\frac{-3}{4}\right| = \frac{3}{4}$

RHS $= |-x| = \left|-\left(\frac{-3}{4}\right)\right| = \left|\frac{3}{4}\right| = \frac{3}{4} =$ RHS

Hence, LHS $=$ RHS

(c) $x = \frac{-4}{-3} = \frac{4}{3}$

LHS $= |x| = \left|\frac{4}{3}\right| = \frac{4}{3}$, RHS $= |-x| = \left|-\frac{4}{3}\right| = \frac{4}{3}$

Hence, LHS $=$ RHS

9. $|x+y| \le |x| + |y|$

(a) $x = \frac{2}{5}, y = -\frac{1}{3}$

$$\text{LHS} = \left|\frac{2}{5}+\left(-\frac{1}{3}\right)\right| = \left|\frac{2}{5}-\frac{1}{3}\right| = \left|\frac{6-5}{15}\right| = \left|\frac{1}{15}\right| = \frac{1}{15}$$

$$\text{RHS} = |x|+|y| = \left|\frac{2}{5}\right|+\left|-\frac{1}{3}\right| = \frac{2}{5}-\frac{1}{3} = \frac{6-5}{15} = \frac{1}{15}$$

Hence, LHS = RHS

(b) $x = \frac{-7}{5}$, $y = \frac{-5}{7}$

$$|x+y| = \left|\frac{-7}{5}+\left(\frac{-5}{7}\right)\right| = \left|-\frac{7}{5}-\frac{5}{7}\right| = \left|\frac{-49-25}{35}\right| = \left|\frac{-74}{35}\right| = \frac{74}{35}$$

$$\text{RHS} = |x|+|y| = \left|\frac{-7}{5}\right|+\left|\frac{-5}{7}\right| = \frac{7}{5}+\frac{5}{7} = \frac{49+25}{35} = \frac{74}{35}$$

Hence, LHS = RHS

10. (a) $|x-y| = |7-3| = |4| = 4$ $\quad x = 7,\ y = 3$ (Given)

and $|y-x| = |3-7| = |-4| = 4$

Hence, $|x-y|$ and $|y-x|$ are equal.

(b) $|x+y| = |-8+2| = |-6| = 6$

Exercise 1.2

1. (a) $\frac{6}{-13}+\frac{5}{7} = -\frac{6}{13}+\frac{5}{7} = \frac{-6\times7+5\times13}{91} = \frac{-42+65}{91} = \frac{23}{91}$

(b) $\frac{-7}{12}+\left(\frac{-11}{-18}\right) = \frac{-7}{12}+\frac{11}{18} = \frac{-7\times3+11\times2}{36} = \frac{-21+22}{36} = \frac{1}{36}$

(c) $5+\frac{6}{15} = \frac{5\times15+6}{15} = \frac{75+6}{15} = \frac{81}{15} = \frac{27}{5}$

2. (a) $\left(\frac{-4}{5}\right)-\left(\frac{6}{15}\right) = -\frac{4}{5}-\frac{6}{15} = \frac{-4\times3-6}{15} = \frac{-12-6}{15} = \frac{-18}{15} = \frac{-6}{5}$

(b) $\left(\frac{-8}{7}\right)-\left(\frac{-5}{13}\right) = \frac{-8}{7}+\frac{5}{13} = \frac{-8\times13+7\times5}{91} = \frac{-104+35}{91} = -\frac{69}{91}$

(c) $\frac{11}{20}-\left(-\frac{13}{14}\right) = \frac{11}{20}+\frac{13}{14} = \frac{11\times7+13\times10}{140} = \frac{77+130}{140} = \frac{207}{140}$

3. (a) $\frac{-7}{9}-\left(\frac{-5}{12}\right)+\frac{1}{3} = -\frac{7}{9}+\frac{5}{12}+\frac{1}{3}$

$$= \frac{-7\times4+5\times3+12}{36} = \frac{-28+15+12}{36} = \frac{-28+27}{36} = -\frac{1}{36}$$

(b) $\frac{11}{18}+\left(\frac{-2}{9}\right)-\frac{3}{16} = \frac{11}{18}-\frac{2}{9}-\frac{3}{16}$

$$= \frac{11\times8-2\times16-3\times9}{144} = \frac{88-32-27}{144} = \frac{88-59}{144} = \frac{29}{144}$$

(c) $\frac{5}{7}-\frac{11}{6}+\frac{8}{9} = \frac{5\times18-11\times21+8\times14}{126}$

$$= \frac{90-231+112}{126} = \frac{202-231}{126} = \frac{-29}{126}$$

4. (a) The additive inverse of $\frac{-6}{13}$ is $\frac{6}{13}$. (b) The additive inverse of $\frac{4}{15}$ is $\frac{-4}{15}$.

(c) The additive inverse of -8 is 8. (d) The additive inverse of $\frac{-16}{-31}$ is $\frac{-16}{31}$.

5. $(x+y)+z=x+(y+z)$

(a) $x=\frac{-1}{2}, y=\frac{5}{12}$ and $z=\frac{-6}{15}$ (Given)

$$\begin{aligned}\text{LHS} &= (x+y)+z\\ &= \left(-\frac{1}{2}+\frac{5}{12}\right)+\left(\frac{-6}{15}\right)\\ &= \left(\frac{-6+5}{12}\right)+\left(\frac{-6}{15}\right)\\ &= -\frac{1}{12}-\frac{6}{15}\\ &= \frac{-5-6\times 4}{60}\\ &= \frac{-5-24}{60}\\ &= \frac{-29}{60}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= x+(y+z)\\ &= \left(-\frac{1}{2}\right)+\left(\frac{5}{12}+\left(\frac{-6}{15}\right)\right)\\ &= \left(-\frac{1}{2}\right)+\left(\frac{5}{12}-\frac{6}{15}\right)\\ &= -\frac{1}{2}+\left(\frac{5\times 5-6\times 4}{60}\right)\\ &= -\frac{1}{2}+\left(\frac{25-24}{60}\right)\\ &= -\frac{1}{2}+\frac{1}{60}\\ &= \frac{-30+1}{60}=\frac{-29}{60}\end{aligned}$$

Hence, LHS = RHS

(b) $x=3,\ y=\frac{-2}{5}$ and $z=\frac{-7}{10}$ (Given)

$$\begin{aligned}\text{LHS} &= (x+y)+z\\ &= \left(3+\left(-\frac{2}{5}\right)\right)+\left(-\frac{7}{10}\right)\\ &= \left(3-\frac{2}{5}\right)+\left(-\frac{7}{10}\right)\\ &= \left(\frac{3\times 5-2\times 1}{5}\right)+\left(-\frac{7}{10}\right)\\ &= \frac{15-2}{5}-\frac{7}{10}\\ &= \frac{13}{5}-\frac{7}{10}\\ &= \frac{13\times 2-7}{10}=\frac{26-7}{10}=\frac{19}{10}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= x+(y+z)\\ &= 3+\left[\left(\frac{-2}{3}\right)+\left(\frac{-7}{10}\right)\right]\\ &= 3+\left(-\frac{2}{5}-\frac{7}{10}\right)\\ &= 3+\left(\frac{-2\times 2-7}{10}\right)\\ &= 3+\left(\frac{-4-7}{10}\right)\\ &= 3-\frac{11}{10}=\frac{3\times 10-11}{10}\\ &= \frac{30-11}{10}=\frac{19}{10}\end{aligned}$$

Hence, LHS = RHS

The property is associative of additive.

6. (a)
$$\begin{aligned}\frac{-6}{5}+\frac{3}{14}+\frac{-6}{7}+\frac{7}{15} &= \left[-\frac{6}{5}+\frac{7}{15}\right]+\left[\frac{3}{14}+\frac{-6}{7}\right]\\ &= \left[\frac{-6\times 3+7}{15}\right]+\left[\frac{3\times 1+2\times(-6)}{14}\right]\end{aligned}$$

$$=\left[\frac{-18+7}{15}\right]+\left[\frac{3-12}{14}\right]$$

$$=\left(\frac{-11}{15}\right)+\left(\frac{-9}{14}\right)$$

$$=\frac{-11}{15}-\frac{9}{14}=\frac{-11\times 14-9\times 15}{210}=\frac{-154-135}{210}=\frac{-289}{210}$$

(b) $-\frac{1}{2}+\frac{7}{4}+\frac{-5}{6}+\frac{11}{6}=\left[-\frac{1}{2}+\frac{7}{4}\right]+\left[\frac{-5}{6}+\frac{11}{6}\right]$

$$=\left[\frac{-2+7}{4}\right]+\left[\frac{-5+11}{6}\right]$$

$$=\frac{5}{4}+\frac{6}{6}=\frac{5}{4}+1$$

$$=\frac{5+4}{4}=\frac{9}{4}=2\frac{1}{4}$$

(c) $\frac{7}{9}+\frac{-2}{3}+\frac{-11}{18}+\frac{1}{6}=\left[\frac{7}{9}+\frac{1}{6}\right]+\left[\frac{-2}{3}+\frac{-11}{18}\right]$

$$=\left[\frac{7\times 2+3}{18}\right]+\left[\frac{-2\times 6+(-11)}{18}\right]$$

$$=\left[\frac{14+3}{18}\right]+\left[\frac{-12-11}{18}\right]=\frac{17}{18}-\frac{23}{18}=\frac{17-23}{18}=\frac{-6}{18}=\frac{-1}{3}$$

7. Sum of two rational numbers $=\frac{21}{8}$

One number $=\frac{-2}{5}$

Other number $=x$

So, $\frac{-2}{5}+x=\frac{21}{8}$

$$x=\frac{21}{8}+\frac{2}{5}$$

$$x=\frac{21\times 5+2\times 8}{40}$$

$$x=\frac{105+16}{40}=\frac{121}{40}$$

Hence, the other rational number is $\frac{121}{40}$.

8. Sum of two rational numbers $=\frac{-15}{16}$

One number $=\frac{-7}{12}$

Other number $=x$

So, $-\frac{7}{12}+x=\frac{-15}{16}$

$$x=-\frac{15}{16}+\frac{7}{12}$$

$$= \frac{-15\times 3+7\times 4}{48}$$

$$x = \frac{-45+28}{48}$$

$$x = \frac{-17}{48}$$

Hence, the other rational number is $\frac{-17}{48}$.

9. Let x be added to $\frac{-3}{8}$ to make the sum $\frac{7}{9}$.

So, $$x+\left(\frac{-3}{8}\right) = \frac{7}{9}$$

$$x = \frac{7}{9}+\frac{3}{8}$$

$$= \frac{7\times 8+3\times 9}{72} = \frac{56+27}{72} = \frac{83}{72}$$

Hence, $\frac{83}{72}$ be added to $\frac{-3}{8}$ to make the sum $\frac{7}{9}$.

10. Let x be subtracted from $\frac{-9}{2}$ to get 1.

So, $$\frac{-9}{2}-x = 1$$

$$\frac{-9}{2}-1 = x$$

$$x = \frac{-9-2}{2} = \frac{-11}{2}$$

Hence, $\frac{-11}{2}$ should be subtracted from $\frac{-9}{2}$ to get 1.

11. $$\left[\left(\frac{3}{-5}\right)+\left(\frac{8}{15}\right)\right]-\left[\left(\frac{-5}{8}\right)+\left(\frac{7}{10}\right)\right]=\left[\frac{-3}{5}+\frac{8}{15}\right]-\left[\frac{-5}{8}+\frac{7}{10}\right]$$

$$=\left(\frac{-3\times 3+8\times 1}{15}\right)-\left(\frac{-5\times 10+7\times 8}{80}\right)$$

$$=\left(\frac{-9+8}{15}\right)-\left(\frac{-50+56}{80}\right)$$

$$=-\frac{1}{15}-\frac{6}{80}=\frac{-16-6\times 3}{240}=\frac{-16-18}{240}=\frac{-34}{240}=\frac{-17}{120}$$

12. Let x be added to $\left(\frac{-5}{7}+\frac{3}{8}\right)$ to obtain $\frac{-9}{4}$.

$$x+\left[\frac{-5}{7}+\frac{3}{8}\right] = \frac{-9}{4}$$

$$x = \frac{-9}{4}+\frac{5}{7}-\frac{3}{8}$$

$$x = \frac{-9\times 14+5\times 8-3\times 7}{56}$$

$$x = \frac{-126+40-21}{56}$$

$$x = \frac{40-147}{56}$$

$$= -\frac{107}{56}$$

Hence, $\frac{-107}{56}$ be added to $\left(\frac{-5}{7}+\frac{3}{8}\right)$ to obtain $\frac{-9}{4}$.

13. The total length of wire $= \frac{15}{4}$ m

One piece $= 2\frac{1}{2}$ m $= \frac{5}{2}$ m

Other piece $= x$

So, $x+\frac{5}{2} = \frac{15}{4}$ m

$$x = \left(\frac{15}{4}-\frac{5}{2}\right) \text{m}$$

$$x = \frac{15-10}{4} \text{m}$$

$$= \frac{5}{4} \text{m} = 1\frac{1}{4} \text{m}$$

Hence, the length of other piece is $1\frac{1}{4}$ m.

14. The total time of a T.V. show with advertisement $= 2\frac{1}{2}$ hours

Advertisement time $= 1\frac{1}{4}$ hours

So, actual show time $= \left(2\frac{1}{2}-1\frac{1}{4}\right)$ hours

$$= \left(\frac{5}{2}-\frac{5}{4}\right) \text{hours}$$

$$= \left(\frac{10-5}{4}\right) \text{hours}$$

$$= \frac{5}{4} \text{hours} = 1\frac{1}{4} \text{hours}$$

Exercise 1.3

1. (a) The multiplicative inverse of $\frac{11}{29}$ is $\frac{29}{11}$.

(b) The multiplicative inverse of $\frac{-16}{23}$ is $\frac{-23}{16}$.

(c) The multiplicative inverse of -13 is $-\frac{1}{13}$.

(d) The multiplicative inverse of $2\frac{1}{3}$ or $\frac{7}{3}$ is $\frac{3}{7}$.

2. (a) $-\frac{4}{5}\times\frac{5}{7}\times\left(-\frac{8}{9}\right)+\frac{8}{9}\times\frac{4}{7}=-\frac{4}{7}\times\left(\frac{-8}{9}\right)+\frac{32}{63}=\frac{32}{63}+\frac{32}{63}=\frac{32+32}{63}=\frac{64}{63}$

(b) $\frac{2}{13}\times\left(-\frac{5}{7}+\frac{1}{7}+\frac{4}{7}\right)=\frac{2}{13}\times\left(\frac{-5+1+4}{7}\right)=\frac{2}{13}\times\left(\frac{-5+5}{7}\right)=\frac{2}{13}\times 0=0$

(c) $\left(-\frac{3}{13}\right)\times\left(-\frac{2}{13}\right)\times\left(-\frac{1}{13}\right)\times 0\times\frac{1}{13}\times\frac{2}{13}\times\frac{3}{13}=0\times\frac{1}{13}\times\frac{2}{13}\times\frac{3}{13}=0$

3. (a) $\frac{3}{7}\times 2\frac{1}{3}\times\frac{3}{7}\times 1\frac{2}{3}=\frac{3}{7}\times\frac{7}{3}\times\frac{3}{7}\times\frac{5}{3}=\frac{5}{7}$

(b) $\frac{4}{7}\times\frac{10}{9}-\frac{4}{7}\times\frac{1}{9}=\frac{4}{7}\left(\frac{10}{9}-\frac{1}{9}\right)$

[By distributive property of multiplication over subtraction]

$$=\frac{4}{7}\left(\frac{10-1}{9}\right)=\frac{4}{7}\times\frac{9}{9}=\frac{4}{7}$$

(c) $\frac{-4}{5}\times\frac{5}{8}+\left(\frac{4}{5}\right)\times\left(-\frac{7}{8}\right)$

$$=\frac{4}{5}\left[\left(\frac{-5}{8}\right)+\left(\frac{-7}{8}\right)\right]$$

[By distributive property of multiplication over addition]

$$=\frac{4}{5}\left[\frac{-5-7}{8}\right]=\frac{4}{5}\left[\frac{-12}{8}\right]=\frac{4}{5}\times\left(\frac{-12}{8}\right)=\frac{-6}{5}$$

(d) $\frac{5}{11}\times\left(\frac{-6}{11}\right)+(-4)\times\frac{5}{11}$

$$=\frac{5}{11}\times\left[\left(\frac{-6}{11}\right)+(-4)\right]$$

[By distributive property of multiplication over addition]

$$=\frac{5}{11}\times\left[\frac{-6+(-4\times 11)}{11}\right]=\frac{5}{11}\times\left[\frac{-6-44}{11}\right]=\frac{5}{11}\times\left(\frac{-50}{11}\right)=\frac{-250}{121}$$

4. (a) $\frac{-7}{9}\times\left(\frac{-14}{25}\times\frac{15}{16}\right)=\left(\frac{-7}{9}\times\frac{-14}{25}\right)\times\frac{15}{16}$

$\text{LHS}=\frac{-7}{9}\times\left(\frac{-7}{5}\times\frac{3}{8}\right)=\frac{49\times 3}{9\times 5\times 8}=\frac{49}{3\times 40}=\frac{49}{120}$

$\text{RHS}=\left(\frac{-7}{9}\times\frac{-14}{25}\right)\times\frac{15}{16}=\frac{7\times 14\times 15}{9\times 25\times 16}=\frac{7\times 7\times 3}{9\times 5\times 8}=\frac{49}{120}$

$\therefore$ LHS $=$ RHS

Hence, $\frac{-7}{9}\times\left(\frac{-14}{25}\times\frac{15}{16}\right)=\left(\frac{-7}{9}\times\frac{-14}{25}\right)\times\frac{15}{16}$ by associative property of multiplication.

(b) $\left(\frac{-1}{21}\times\frac{14}{15}\right)\times\frac{25}{-26}=\frac{-1}{21}\times\left(\frac{14}{15}\times\frac{25}{-26}\right)$

$$\text{LHS} = \left(\frac{-1}{21}\times\frac{14}{15}\right)\times\frac{25}{-26} = \left(\frac{-1}{3}\times\frac{2}{15}\right)\times\frac{25}{-26} = \left(\frac{-2}{45}\right)\times\left(\frac{25}{-26}\right) = \frac{5}{117}$$

$$\text{RHS} = -\frac{1}{21}\times\left(\frac{14}{15}\times\frac{25}{-26}\right) = -\frac{1}{21}\times\left(\frac{7\times5}{3\times-13}\right) = -\frac{1}{21}\times\frac{35}{-39} = +\frac{5}{3\times39} = \frac{5}{117}$$

$\therefore$ LHS = RHS

Hence, $\left(-\frac{1}{21}\times\frac{14}{15}\right)\times\frac{25}{-26} = -\frac{1}{21}\times\left(\frac{14}{15}\times\frac{25}{-26}\right)$ by associative property of multiplication.

(c) $-56\times\frac{2}{7} = \frac{2}{7}\times(-56)$

$\text{LHS} = -56\times\frac{2}{7} = -8\times2 = -16$ $\quad$ $\text{RHS} = \frac{2}{7}\times(-56) = 2\times(-8) = -16$

$\therefore$ LHS = RHS

Hence, $-56\times\frac{2}{7} = \frac{2}{7}\times(-56)$ by commutative property of multiplication.

5. (a) $\frac{-3}{8}\times\left(\frac{-6}{11}+\frac{4}{9}\right) = \left(\frac{-3}{8}\times\frac{-6}{11}\right)+\left(\frac{-3}{8}\times\frac{4}{9}\right)$

$$\text{LHS} = \frac{-3}{8}\times\left(\frac{-6}{11}+\frac{4}{9}\right)$$

$$= \frac{-3}{8}\times\left(\frac{-6\times9+4\times11}{99}\right)$$

$$= \frac{-3}{8}\times\left(\frac{-54+44}{99}\right) = -\frac{3}{8}\times\left(\frac{-10}{99}\right) = \frac{5}{4\times33} = \frac{5}{132}$$

$$\text{RHS} = \left(\frac{-3}{8}\times\frac{-6}{11}\right)+\left(\frac{-3}{8}\times\frac{4}{9}\right)$$

$$= \left(\frac{3\times3}{4\times11}\right)+\left(\frac{-1}{2\times3}\right) = \frac{9}{44}-\frac{1}{6} = \frac{27-22}{132} = \frac{5}{132}$$

$\therefore$ LHS = RHS

Hence, $\frac{-3}{8}\times\left(\frac{-6}{11}+\frac{4}{9}\right) = \left(\frac{-3}{8}\times\frac{-6}{11}\right)+\left(\frac{-3}{8}\times\frac{4}{9}\right)$

by distributive property of multiplication over addition.

(b) $\frac{8}{7}\times\left(\frac{-10}{9}+\frac{-1}{3}\right) = \left(\frac{8}{7}\times\frac{-10}{9}\right)+\left(\frac{8}{7}\times\frac{-1}{3}\right)$

$$\text{LHS} = \frac{8}{7}\times\left(\frac{-10}{9}+\frac{-1}{3}\right) = \frac{8}{7}\times\left(\frac{-10-3}{9}\right)$$

$$= \frac{8}{7}\times\left(-\frac{13}{9}\right) = \frac{-104}{63}$$

$$\text{RHS} = \left(\frac{8}{7}\times\frac{-10}{9}\right)+\left(\frac{8}{7}\times\frac{-1}{3}\right) = \frac{-80}{63}+\frac{-8}{21}$$

$$= \frac{-80-8\times3}{63} = \frac{-80-24}{63} = \frac{-104}{63}$$

$\therefore$ LHS = RHS

Hence, $\frac{8}{7}\times\left(\frac{-10}{9}+\frac{-1}{3}\right)=\left(\frac{8}{7}\times\frac{-10}{9}\right)+\left(\frac{8}{7}\times\frac{-1}{3}\right)$

by distributive property of multiplication over addition.

(c) $\frac{-8}{9}\times\frac{-13}{14}=\frac{-13}{14}\times\frac{-8}{9}$

$\text{LHS}=\frac{-8}{9}\times\frac{-13}{14}=\frac{4\times13}{9\times7}=\frac{52}{63}$ $\quad$ $\text{RHS}=\frac{-13}{14}\times\frac{-8}{9}=\frac{13\times4}{7\times9}=\frac{52}{63}$

$\therefore$ LHS = RHS

Hence, commutative property of multiplication.

6. The cost of one litre of petrol $=₹\,42\frac{3}{7}=₹\,\frac{297}{7}$

$\therefore$ the cost of 28 litres of petrol $=₹\,\frac{297}{7}\times28$

$=₹\,297\times4$

$=₹\,1188$

7. The length of a rectangular park $=15\frac{2}{5}\text{ m}=\frac{77}{5}\text{ m}$

And breadth $=9\frac{1}{6}\text{ m}=\frac{55}{6}\text{ m}$

$\because$ the area of a rectangular park = length × breadth

$=\frac{77}{5}\times\frac{55}{6}\text{ m}^2$

$=\frac{77\times11}{6}\text{ m}^2=\frac{847}{6}\text{ m}^2$

$=141\frac{1}{6}\text{ m}^2$

Hence, the area of a rectangular park is $141\frac{1}{6}\text{ m}^2$.

8. The side of a square piece of land $=6\frac{1}{4}\text{ m}$

$\therefore$ the area of a square piece of lands. $=\text{side}^2$

$=\left(6\frac{1}{4}\right)\text{m}^2$

$=\left(\frac{25}{4}\right)\text{m}^2=\frac{625}{16}\text{ m}^2=39\frac{1}{16}\text{ m}^2$

Hence, the area of a square piece of land is $39\frac{1}{16}\text{ m}^2$.

Exercise 1.4

1. (a) $\frac{-36}{55}\div\frac{-3}{10}=\frac{-36}{55}\times\left(-\frac{10}{3}\right)=\frac{36}{55}\times\frac{10}{3}=\frac{12}{55}\times10=\frac{12\times2}{11}=\frac{24}{11}=2\frac{2}{11}$

(b) $\frac{-2}{21}\div\frac{7}{-14}=+\left(\frac{2}{21}\times\frac{14}{7}\right)=\frac{2\times2}{21}=\frac{4}{21}$

(c) $-7 \div \frac{5}{13} = -\left(7 \times \frac{13}{5}\right) = -\frac{91}{5} = -18\frac{1}{5}$

(d) $-2\frac{3}{8} \div \frac{35}{42} = \frac{-19}{8} \times \frac{42}{35} = \frac{-19 \times 21}{4 \times 35} = \frac{-19 \times 3}{4 \times 5} = \frac{-57}{20}$

2. The product of two rational numbers $= \frac{-56}{85}$

One of the number $= \frac{35}{-63}$

Other number $= ?$

So, other number $= \frac{-56}{85} \div \left(\frac{-35}{63}\right)$

$= \frac{56}{85} \times \frac{63}{35} = \frac{8 \times 63}{85 \times 5} = \frac{504}{425}$

Hence, the other number is $\frac{504}{425}$.

3. The product of two rational numbers $= \frac{-72}{121}$

One of the number $= \frac{88}{27}$

Other number $= ?$

So, Other number $= \frac{-72}{121} \div \frac{88}{27}$

$= -\frac{72}{121} \times \frac{27}{88} = \frac{-9 \times 27}{121 \times 11}$

$= \frac{-243}{1331}$

Hence, the other number is $\frac{-243}{1331}$.

4. Let x be multiplied by $\frac{-12}{13}$ to get $\frac{4}{39}$

Then, $x \times \left(\frac{-12}{13}\right) = \frac{4}{39}$

$x = \frac{4}{39} \times \frac{13}{-12}$

$= \frac{1}{3 \times (-3)} = -\frac{1}{9}$

Hence, $\frac{-1}{9}$ should be multiplied by $\frac{-12}{13}$ to get $\frac{4}{39}$.

5. Let $\frac{-54}{15}$ should be divided by x to get $\frac{-42}{35}$.

Then, $\frac{-54}{15} \div x = \frac{-42}{35}$

$$\frac{-54}{15} \times \frac{1}{x} = \frac{-42}{35}$$

$$x = +\frac{54 \times 35}{15 \times 42}$$

$$= \frac{1890}{630} = 3$$

Hence, $\frac{-54}{15}$ should be divided by 3 to get $\frac{-42}{35}$.

6. $\left[\frac{11}{7} + \frac{-7}{5}\right] \div \left[\frac{11}{7} \times \frac{-7}{5}\right] = \left[\frac{11 \times 5 - 49}{35}\right] \div \left[\frac{-11}{5}\right]$

$$= \frac{55 - 49}{35} \times \left(\frac{-5}{11}\right) = -\frac{6}{35} \times \frac{5}{11} = \frac{-6}{7 \times 11} = \frac{-6}{77}$$

7. $\left[\frac{-9}{4} + \frac{-8}{3}\right] \div \left[\frac{13}{8} - \frac{-7}{16}\right] = \left[\frac{-9 \times 3 + (-8 \times 4)}{12}\right] \div \left[\frac{13 \times 2 + 7}{16}\right]$

$$= \left[\frac{-27 - 32}{12}\right] \div \left[\frac{26 + 7}{16}\right]$$

$$= \frac{-59}{12} \div \frac{33}{16} = \frac{-59}{12} \times \frac{16}{33} = \frac{-59 \times 4}{3 \times 33} = \frac{-236}{99}$$

8. Let the required number be *x*.

(a)

$$\frac{7}{12} \div x = \frac{-14}{3}$$

$$\frac{7}{12} \times \frac{1}{x} = \frac{-14}{3}$$

$$x = \frac{-7 \times 3}{12 \times 14}$$

$$x = -\frac{1}{4 \times 2} = -\frac{1}{8}$$

(b)

$$x \div \left(-\frac{6}{5}\right) = \frac{15}{26}$$

$$x \times \left(\frac{-5}{6}\right) = \frac{15}{26}$$

$$x = \frac{-6 \times 15}{26 \times 5}$$

$$x = \frac{-6 \times 3}{26}$$

$$x = \frac{-9}{13}$$

(c)

$$-25 \div x = -\frac{5}{6}$$

$$-25 \times \frac{1}{x} = -\frac{5}{6}$$

$$x = -25 \times \left(-\frac{6}{5}\right)$$

$$x = 5 \times 6$$
$$x = 30$$

(d) $$x \div (-13) = -\frac{4}{11}$$
$$x \times \left(\frac{-1}{13}\right) = \frac{-4}{11}$$
$$x = +\frac{4 \times 13}{11}$$
$$x = \frac{52}{11}$$

9. The area of a rectangle $= 45\frac{1}{4}\ \text{m}^2 = \frac{181}{4}\ \text{m}^2$

length $= 25\frac{3}{8}\ \text{m} = \frac{203}{8}\ \text{m}$

breadth $= ?$

$\because$ breadth $=$ the area of a rectangle $\div$ length

$$= \left(\frac{181}{4} \div \frac{203}{8}\right)\text{m}$$
$$= \frac{181}{4} \times \frac{8}{203}\ \text{m} = \frac{362}{203}\ \text{m or } 1\frac{159}{203}\ \text{m}$$

Hence, the breadth of a rectangle is $1\frac{159}{203}$ m.

10. The cost of $5\frac{2}{7}$ m or $\frac{37}{7}$ m of cloth $=$ ₹ $28\frac{1}{3} =$ ₹ $\frac{85}{3}$

$\therefore$ the cost of one metre of cloth $=$ ₹ $\left(\frac{85}{3} \div \frac{37}{7}\right)$

$$= ₹\left(\frac{85}{3} \times \frac{7}{37}\right) = ₹\,\frac{595}{111}$$

11. Total length of the wire $= 64\frac{1}{6}\ \text{m} = \frac{385}{6}\ \text{m}$

The number of wire pieces that can be cut $= 5$

$\therefore$ the required length of each wire $= \left(\frac{385}{6} \div 5\right)\text{m}$

$$= \left(\frac{385}{6} \times \frac{1}{5}\right)\text{m} = \frac{77}{6}\ \text{m} = 12\frac{5}{6}\ \text{m}$$

Hence, the required length of each wire is $12\frac{5}{6}$ m.

12. (a) $$\frac{-13}{25} \div \frac{4}{-19} = \frac{-4}{19} \div \frac{-13}{25}$$

$$\text{LHS} = \frac{-13}{25} \div \frac{4}{-19} = \frac{+13}{25} \times \frac{19}{4} = \frac{247}{100}$$

$$\text{RHS} = \frac{-4}{19} \div \frac{-13}{25} = +\frac{4}{19} \times \frac{25}{13} = \frac{100}{247}$$

$\therefore$ LHS $\neq$ RHS

Hence, the statement are not true.

(b) $\left(\frac{4}{7}\div\frac{2}{9}\right)\div\frac{11}{13}=\frac{4}{7}\div\left(\frac{2}{9}\div\frac{11}{13}\right)$

$$\text{LHS}=\left(\frac{4}{7}\div\frac{2}{9}\right)\div\frac{11}{13}=\left(\frac{4}{7}\times\frac{9}{2}\right)\times\frac{13}{11}=\left(\frac{2\times 9}{7}\right)\times\frac{13}{11}=\frac{18\times 13}{7\times 11}=\frac{234}{77}$$

$$\text{RHS}=\frac{4}{7}\div\left(\frac{2}{9}\div\frac{11}{13}\right)=\frac{4}{7}\div\left(\frac{2}{9}\times\frac{13}{11}\right)=\frac{4}{7}\times\frac{9\times 11}{2\times 13}=\frac{2\times 9\times 11}{7\times 13}=\frac{198}{91}$$

$\therefore$ LHS $\neq$ RHS

Hence, the statement are false.

(c) $\frac{5}{-13}\div\frac{-2}{7}=\frac{-2}{7}\times\frac{5}{-13}$

$\text{LHS}=\frac{5}{-13}\div\frac{-2}{7}=+\frac{5}{13}\times\frac{7}{2}=\frac{35}{26}$ $\qquad$ $\text{RHS}=\frac{-2}{7}\times\frac{5}{-13}=+\frac{10}{91}$

$\therefore$ LHS $\neq$ RHS

Hence, the statement are false.

(d) $\left[(-17)\div\frac{8}{5}\right]\div\frac{-9}{19}=(-17)\div\left[\frac{8}{5}\div\frac{-9}{19}\right]$

$$\text{LHS}=\left[(-17)\div\frac{8}{5}\right]\div\frac{-9}{19}=\left[(-17)\times\frac{5}{8}\right]\div\frac{-9}{19}=+\frac{85}{8}\times\frac{19}{9}=\frac{1615}{72}$$

$$\text{RHS}=(-17)\div\left[\frac{8}{5}\div\frac{-9}{19}\right]=(-17)\div\left[\frac{8}{5}\times\frac{19}{-9}\right]=(-17)\div\left(\frac{152}{-45}\right)$$

$$=\frac{+17\times 45}{152}=\frac{765}{152}$$

$\therefore$ LHS $\neq$ RHS

Hence, the statement are false.

Exercise 1.5

1. The first rational number $=\frac{1}{2}\left[\frac{-2}{7}+\frac{2}{7}\right]=\frac{1}{2}[0]=0$

The second rational number $=\frac{1}{2}\left[\frac{-2}{7}+0\right]=-\frac{1}{7}$

And the third rational number $=\frac{1}{2}\left[0+\frac{2}{7}\right]=\frac{1}{7}$

Hence, $-\frac{1}{7}, 0\ \frac{1}{7}$ are three rational numbers between $\frac{-2}{7}$ and $\frac{2}{7}$.

2. LCM of 3 and 9 is 9.

Hence, $\frac{-1}{3}=\frac{-1\times 3}{3\times 3}=\frac{-3}{9}$ and $\frac{4}{9}=\frac{4\times 1}{9\times 1}=\frac{4}{9}$

Now, 4 rational numbers between $\frac{-1}{3}$ and $\frac{4}{9}$ or $\frac{-3}{9}$ and $\frac{4}{9}$ are $\frac{-2}{9}, \frac{-1}{9}, 0, \frac{1}{9}$.

3. A rational number between 3 and 4 $=\frac{1}{2}[3+4]=\frac{7}{2}$

4. LCM of 9 and 8 is 72.

Hence, $\frac{2}{9}=\frac{2\times 8}{9\times 8}=\frac{16}{72}$ and $\frac{5}{8}=\frac{5\times 9}{8\times 9}=\frac{45}{72}$

Now, six rational numbers between $\frac{2}{9}$ and $\frac{5}{8}$ or $\frac{16}{72}$ and $\frac{45}{72}$ are :

$\frac{17}{72},\frac{18}{72},\frac{19}{72},\frac{20}{72},\frac{21}{72}$ and $\frac{22}{72}$

or $\frac{17}{72},\frac{1}{4},\frac{19}{72},\frac{5}{18},\frac{7}{24},\frac{11}{36}.$

5. LCM of 4 and 8 is 8.

Hence, $-\frac{1}{4}=-\frac{2}{8}$ and $\frac{3}{8}=\frac{3\times 1}{8\times 1}=\frac{3}{8}$

Now, 4 rational numbers between $-\frac{1}{4}$ and $\frac{3}{8}$ or $\frac{-2}{8}$ and $\frac{3}{8}$ are :

$-\frac{1}{8}$, 0, $\frac{1}{8}$ and $\frac{2}{8}$ or $\frac{1}{4}$

$\therefore$ The fifth rational number $=\frac{1}{2}\left[0+\frac{1}{8}\right]=\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}$

Hence, five rational numbers are $-\frac{1}{8}$, 0, $\frac{1}{16}$, $\frac{1}{8}$ and $\frac{1}{4}$.

6. LCM of 3 and 5 is 15.

Hence, $\frac{2}{3}=\frac{2\times 5}{3\times 5}=\frac{10}{15}$ and $\frac{2}{5}=\frac{2\times 3}{5\times 3}=\frac{6}{15}$

Also, $\frac{10}{15}=\frac{100}{150}$ and $\frac{6}{15}=\frac{60}{150}$

Hence, ten rational numbers between $\frac{2}{3}$ and $\frac{2}{5}$ or $\frac{100}{150}$ and $\frac{60}{150}$ are :

$\frac{61}{150},\frac{62}{150},\frac{63}{150},\frac{64}{150},\frac{65}{150},\frac{66}{150},\frac{67}{150},\frac{68}{150},\frac{69}{150}$ and $\frac{70}{150}$

or $\frac{61}{150},\frac{31}{75},\frac{21}{50},\frac{32}{75},\frac{13}{30},\frac{22}{50},\frac{67}{150},\frac{34}{75},\frac{23}{50}$ and $\frac{7}{15}.$

7. (a) $x=\frac{-3}{8}$ and $|x|=\left|\frac{-3}{8}\right|=\frac{3}{8}$

Now, 5 rational numbers between $\frac{-3}{8}$ and $\frac{3}{8}$ are :

$\frac{-2}{8},\frac{-1}{8},0,\frac{1}{8},\frac{2}{8}$

$\therefore$ sixth rational numbers between $\frac{-1}{8}$ and 0

$$=\frac{1}{2}\left[-\frac{1}{8}+0\right]=-\frac{1}{16}$$

seventh rational number between $-\frac{1}{16}$ and 0

$$=\frac{1}{2}\left[-\frac{1}{16}+0\right]=-\frac{1}{32}$$

eight rational number between $\frac{1}{8}$ and 0

$$= \frac{1}{2}\left[\frac{1}{8} + 0\right] = \frac{1}{16}$$

nineth rational number between $\frac{1}{16}$ and $0 = \frac{1}{2}\left[\frac{1}{16} + 0\right] = \frac{1}{32}$

And tenth rational number between $\frac{1}{32}$ and $0 = \frac{1}{2}\left[\frac{1}{32} + 0\right] = \frac{1}{64}$

Hence ten rational numbers between $\frac{-3}{8}$ and $\frac{3}{8}$ are :

$-\frac{2}{8}, -\frac{1}{8}, -\frac{1}{16}, -\frac{1}{32}\ 0\ \frac{1}{64}, \frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{2}{8}$

or $-\frac{1}{4}, -\frac{1}{8}, -\frac{1}{16}, -\frac{1}{32}, 0, \frac{1}{64}, \frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}$

(b) We have, $x = -\frac{5}{7}$

$|x| = \left|-\frac{5}{7}\right| = \frac{5}{7}$

Now, we have to find 9 rational numbers between $-\frac{5}{7}$ and $\frac{5}{7}$.

As $-4, -3, -2, -1\ 0, 1, 2, 3, 4$ lie between -5 and 5, therefore, $\frac{-4}{7}, \frac{-3}{7}, \frac{-2}{7}, \frac{-1}{7}, 0, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}$ and $\frac{4}{7}$ lie between $\frac{-5}{7}$ and $\frac{5}{7}$.

∴ ten rational number between 0 and $\frac{1}{7}$

$$= \frac{1}{2}\left[0 + \frac{1}{7}\right] = \frac{1}{14}$$

Hence, ten rational numbers between $\frac{-5}{7}$ and $\frac{5}{7}$ are :

$$\frac{-4}{7}, \frac{-3}{7}, \frac{-2}{7}, \frac{-1}{7}, 0, \frac{1}{14}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7} \text{ and } \frac{4}{7}.$$

MCQs

1. (d) **2.** (c) **3.** (b) **4.** (b) **5.** (a) **6.** (b)

2 Exponents (Powers)

Exercise 2.1

1. (a) $\frac{-11}{3} \times \frac{-11}{3} = \left(-\frac{11}{2}\right)^2$

(b) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^3$

(c) $\frac{9}{11} \times \frac{9}{11} \times \frac{9}{11} \times \frac{9}{11} = \left(\frac{9}{11}\right)^4$

(d) $\frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} = \left(\frac{-3}{7}\right)^5$

2. (a) $\left(\frac{4}{7}\right)^3 = \frac{4\times4\times4}{7\times7\times7} = \frac{64}{343}$

(b) $\left(\frac{-3}{4}\right)^5 = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{4\times4\times4\times4\times4} = \frac{9\times9\times(-3)}{16\times16\times4} = \frac{243}{1024}$

(c) $\left(\frac{7}{2}\right)^2 = \frac{7\times7}{2\times2} = \frac{14}{4}$

(d) $\left(\frac{3}{5}\right)^3 = \frac{3\times3\times3}{5\times5\times5} = \frac{27}{125}$

(e) $\left(\frac{-4}{5}\right)^4 = \frac{(-4)\times(-4)\times(-4)\times(-4)}{5\times5\times5\times5} = \frac{16\times16}{25\times25} = \frac{256}{625}$

(f) $\left(\frac{-1}{3}\right)^7 = \frac{(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)}{3\times3\times3\times3\times3\times3\times3} = \frac{-1}{2187}$

(g) $\left(\frac{-13}{11}\right)^2 = \frac{(-13)\times(-13)}{11\times11} = \frac{169}{121}$

(h) $\left(\frac{-7}{5}\right)^5 = \frac{(-7)\times(-7)\times(-7)\times(-7)\times(-7)}{5\times5\times5\times5\times5} = \frac{-16807}{3125}$

3. (a) $\frac{49}{25} = \frac{7\times7}{5\times5} = \left(\frac{7}{5}\right)^2$

(b) $\frac{64}{169} = \frac{8\times8}{13\times13} = \frac{8^2}{13^2} = \left(\frac{8}{13}\right)^2$

(c) $\frac{81}{25} = \frac{9\times9}{5\times5} = \frac{9^2}{5^2} = \left(\frac{9}{5}\right)^2$

(d) $\frac{125}{64} = \frac{5\times5\times5}{4\times4\times4} = \frac{5^3}{4^3} = \left(\frac{5}{4}\right)^3$

(e) $\frac{256}{289} = \frac{16\times16}{17\times17} = \frac{16^2}{17^2} = \left(\frac{16}{17}\right)^2$

(f) $\frac{32}{243} = \frac{2\times2\times2\times2\times2}{3\times3\times3\times3\times3} = \frac{2^5}{3^5} = \left(\frac{2}{3}\right)^5$

(g) $-\frac{1}{125} = \frac{(-1)\times(-1)\times(-1)}{5\times5\times5} = -\frac{1}{125}$

(h) $-\frac{27}{52} = \frac{(-3)\times(-3)\times(-3)}{8\times8\times8} = \frac{(-3)^3}{8^3} = \left(\frac{-3}{8}\right)^3$

4. (a) The reciprocal of 9 is $\frac{1}{9}$.

(b) The reciprocal of -3 is $-\frac{1}{3}$.

(c) The reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$.

(d) The reciprocal of $\frac{-11}{3}$ is $\frac{-3}{11}$.

(e) The reciprocal of $\left(\frac{1}{13}\right)^4$ is $(13)^4$.

(f) The reciprocal of $\left(\frac{-19}{7}\right)^2$ is $\left(-\frac{7}{19}\right)^2$.

(g) The reciprocal of $\left(\frac{17}{-15}\right)^3$ is $\left(-\frac{15}{17}\right)^3$.

(h) The reciprocal of $\left(\frac{-8}{-9}\right)^5$ is $\left(\frac{-9}{-8}\right)^5$.

5. (a) $\left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right)^2 = \left(\frac{2\times 2\times 2\times 2}{3\times 3\times 3\times 3}\right)\times\left(\frac{1}{3\times 3}\right) = \frac{16}{9\times 9\times 9} = \frac{16}{729}$

(b) $$\left(\frac{-6}{7}\right)^5 \times \left(\frac{-2}{6}\right)^6 = \frac{(-6)\times(-6)\times(-6)\times(-6)\times(-6)}{7\times 7\times 7\times 7\times 7}$$

$$\times \frac{(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)}{6\times 6\times 6\times 6\times 6\times 6}$$

$$= \frac{(-1)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)}{7\times 7\times 7\times 7\times 7\times 6}$$

$$= -\frac{2\times 2\times 2\times 2\times 2}{49\times 49\times 7\times 3} = \frac{-32}{7203\times 7} = -\frac{32}{50421}$$

(c) $$\left(-\frac{1}{3}\right)^5 \times 2^3 \times \left(\frac{3}{4}\right)^4$$

$$= \frac{(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times 2\times 2\times 2\times 3\times 3\times 3\times 3}{3\times 3\times 3\times 3\times 3\times 4\times 4\times 4\times 4}$$

$$= \frac{-1\times 1\times 1}{3\times 2\times 16} = \frac{-1}{96}$$

(d) $$\left(\frac{-4}{7}\right)^2 \times \left(\frac{3}{4}\right)^3 \times \left(\frac{7}{5}\right)^8 = \frac{(-4)\times(-4)\times 3\times 3\times 3\times 7\times 7\times 7\times 7\times 7\times 7\times 7\times 7}{7\times 7\times 4\times 4\times 4\times 5\times 5\times 5\times 5\times 5\times 5\times 5\times 5}$$

$$= \frac{3\times 3\times 3\times 7\times 7\times 7\times 7\times 7\times 7}{4\times 5\times 5\times 5\times 5\times 5\times 5\times 5\times 5}$$

$$= \frac{27\times 49\times 49\times 49}{4\times 25\times 25\times 25\times 25} = \frac{3176523}{1562500}$$

6. (a) $\left(\frac{3}{11}\right)^8 \div \left(\frac{3}{11}\right)^5 = \left(\frac{3}{11}\right)^{8-5}$ $[\because x^m \div x^n = x^{m-n}]$

$\therefore$ $\left(\frac{3}{11}\right)^3 = \frac{3\times 3\times 3}{11\times 11\times 11} = \frac{27}{1331}$

(b) $\left[\frac{13}{17}\right]^{15} \div \left(\frac{13}{17}\right)^{11} = \left(\frac{13}{17}\right)^{15-11} = \left(\frac{13}{17}\right)^4 = \frac{13\times 13\times 13\times 13}{17\times 17\times 17\times 17} = \frac{28561}{83521}$

(c) $\left(\frac{4}{15}\right)^6 \div \left(\frac{4}{15}\right)^9 = \left(\frac{4}{15}\right)^{6-9} = \left(\frac{4}{15}\right)^{-3} = \left(\frac{15}{4}\right)^3 = \frac{3375}{64}$

(d) $\left(\frac{15}{19}\right)^{11} \div \left(\frac{15}{19}\right)^8 = \left(\frac{15}{19}\right)^{11-8} = \left(\frac{15}{19}\right)^3 = \frac{3375}{6859}$

7. (a) $(-9)^{-1} = \left(-\frac{1}{9}\right) = -\frac{1}{9}$ (b) $\left(-\frac{1}{9}\right)^{-1} = (-9) = -9$

(c) $\left(\frac{7}{13}\right)^{-1} = \frac{13}{7}$

(d) $(-5)^{-1} \times \left(\frac{1}{5}\right)^{-1} = \left(-\frac{1}{5}\right) \times 5 = \frac{-5}{5} = -1$

(e) $\left(\frac{3}{7}\right)^{-1} \times \left(\frac{7}{5}\right)^{-1} = \left(\frac{7}{3}\right) \times \left(\frac{5}{7}\right) = \frac{5}{3}$

(f) $(6^{-1} - 7^{-1})^{-1} = \left(\frac{1}{6} - \frac{1}{7}\right)^{-1} = \left(\frac{7-6}{42}\right)^{-1} = \left(\frac{1}{42}\right)^{-1} = 42$

8. (a) $\left[\left\{\left(\frac{2}{5}\right)^{4}\right\}^{2} \div \left(\frac{2}{5}\right)^{2}\right] \times \left(\frac{1}{2}\right)^{-2} \times 2^{-1} \times \left(\frac{1}{4}\right)^{-1}$

$$= \left[\left(\frac{2}{5}\right)^{8} \div \left(\frac{2}{5}\right)^{2}\right] \times (2)^{2} \times \frac{1}{2} \times 4$$

$$= \left(\frac{2}{5}\right)^{8-2} \times 4 \times \frac{1}{2} \times 4 = \left(\frac{2}{5}\right)^{6} \times 8$$

$$= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{5 \times 5 \times 5 \times 5 \times 5 \times 5} \times 8$$

$$= \frac{64 \times 8}{15625} = \frac{512}{15625}$$

(b) $\left(\frac{9}{5}\right)^{-3} \times \left(\frac{12}{7}\right)^{0} \times 5^{-3} \times \left(\frac{1}{9}\right)^{-1} = \left(\frac{5}{9}\right)^{3} \times 1 \times \left(\frac{1}{5}\right)^{3} \times 9 \quad [\because x^{0} = 1]$

$$= \frac{5 \times 5 \times 5 \times 9}{9 \times 9 \times 9 \times 5 \times 5 \times 5}$$

$$= \frac{1 \times 1}{9 \times 9} = \frac{1}{81}$$

9.
$$\left(\frac{2}{5}\right)^{3} \times \left(\frac{2}{6}\right)^{-6} = \left(\frac{2}{5}\right)^{2a-1}$$

$$\left(\frac{2}{5}\right)^{3+(-6)} = \left(\frac{2}{5}\right)^{2a-1}$$

$$\left(\frac{2}{5}\right)^{-3} = \left(\frac{2}{5}\right)^{2a-1}$$

Since, both base are same.
So, powers can be compared

$$2a - 1 = -3$$
$$2a = -3 + 1$$
$$2a = -2$$
$$a = -2 \div 2$$
$$a = -1$$

10. Let the required number be *x*.
Then,

$$x \times (-3)^{-1} = (7)^{-1}$$
$$x = (7)^{-1} \div (-3)^{-1}$$
$$x = \left(\frac{1}{7}\right) \div \left(\frac{-1}{3}\right)$$
$$x = \left(\frac{1}{7}\right) \times (-3)$$
$$= \frac{-3}{7}$$

Hence, the required number is $-\frac{3}{7}$.

11. Let the required number be *x*.

Then,
$$(-40)^{-1} \div x = 5^{-1}$$
$$\left(-\frac{1}{40}\right) \div x = \frac{1}{5}$$
$$-\frac{1}{40} \times x = \frac{1}{5}$$
$$x = -\frac{1}{5} \times 40$$
$$x = -8$$

Hence, the required number is (-8).

12. Let the required number be *x*.

Then,
$$x \times (2^{-3}) = 5$$
$$x \times \left(\frac{1}{2}\right)^{3} = 5$$
$$x = 5 \times (2)^{3}$$
$$x = 5 \times 8$$
$$= 40$$

Hence, the required number is 40.

13. Let the required number be *x*.

Then,
$$\left(\frac{-2}{9}\right)^{-3} \div x = -\frac{3}{8}$$
$$\left(-\frac{9}{2}\right)^{3} \div x = -\frac{3}{8}$$
$$\left(\frac{-9}{2}\right)^{3} \times \frac{1}{x} = -\frac{3}{8}$$
$$x = +\frac{8}{3} \times \frac{9^3}{2^3}$$
$$x = 9 \times 9 \times 3$$
$$x = 243$$

Hence, the required number is 243.

14. $\frac{x}{y}=\left(\frac{-2}{3}\right)^3 \div \left(\frac{3}{5}\right)^{-2}$ (Given)

(a) $$\left(\frac{x}{y}\right)^{-2} = \left[\left(\frac{-2}{3}\right)^3 \div \left(\frac{3}{5}\right)^{-2}\right]^{-2}$$

$$=\left[\left(\frac{-2}{3}\right)^3 \div \left(\frac{5}{3}\right)^{2}\right]^{-2}$$

$$=\left[\left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^{2}\right]^{-2}$$

$$=\left[\frac{(-2)\times(-2)\times(-2)}{3\times3\times3}\times\frac{3\times3}{5\times5}\right]^{-2}$$

$$=\left(\frac{-8}{3}\times\frac{1}{25}\right)^{-2}=\left(\frac{-8}{75}\right)^{-2}$$

$$=\left(-\frac{75}{8}\right)^{2}=\frac{75\times75}{8\times8}=\frac{5625}{64}$$

(b) $$\left(\frac{x}{y}+\frac{y}{x}\right)^{-1} = \left[\left\{\left(\frac{-2}{3}\right)^3 \div \left(\frac{3}{5}\right)^{-2}\right\}+\left\{\frac{1}{\left(\frac{-2}{3}\right)^3 \div \left(\frac{3}{5}\right)^{-2}}\right\}\right]^{-1}$$

$$=\left[\left\{\left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^{2}\right\}+\left\{\frac{1}{\left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^{2}}\right\}\right]^{-1}$$

$$=\left[\left(\frac{-8}{75}\right)+\frac{1}{\left(\frac{-8}{75}\right)}\right]^{-1}$$

$$=\left[\left(\frac{-8}{75}\right)+\left(\frac{-75}{8}\right)\right]^{-1}=\left[\frac{-8\times8-75\times75}{600}\right]^{-1}$$

$$=\left[\frac{-64-5625}{600}\right]^{-1}=\left[-\frac{5689}{600}\right]^{-1}=-\frac{600}{5689}$$

(c) $$\left(\frac{x}{y}\right)^{-\frac{8y}{5x}} = \left[\left(\frac{-2}{3}\right)^3 \div \left(\frac{3}{5}\right)^{-2}\right]^{-\frac{8}{5}\left[\left(-\frac{2}{3}\right)^3 \div \left(\frac{3}{5}\right)^{-2}\right]^{-1}}$$

$$=\left[\frac{-8}{3}\times\frac{1}{25}\right]^{-\frac{8}{5}\left[\frac{-8}{3}\times\frac{1}{25}\right]^{-1}}$$

$$=\left(\frac{-8}{75}\right)^{-\frac{8}{5}\times\left(\frac{-8}{75}\right)^{-1}}$$

$$=\left(\frac{-8}{75}\right)^{-\frac{8}{5}\times\left(-\frac{75}{8}\right)}$$

$$=\left(\frac{-8}{75}\right)^{15}$$

Exercise 2.2

1. (a) 473 written as 4.73×10^{2}

(b) 68450 written as 6.845×10^{4}

(c) 459731452 written as 4.59731452×10^{8}

(d) 0.31 written as 3.1×10^{-1}

(e) 0.00000879 written as 8.79×10^{-6}

(f) 0.000000000009 written as 9.0×10^{-12}

2. (a) $0.00035 = 3.5 \times 10^{-4}$ (b) $3.0012 = 3.0012 \times 10^{0}$

(c) $0.0001 = 1.000 \times 10^{-4}$ (d) $30085 = 30.085 \times 10^{3}$

(e) $5.3897 = 5.3897 \times 10^{0}$ (f) $0.056739 = 5.6739 \times 10^{-2}$

3. (a) $8.0031 \times 10^{-3} = 0.0080031$ (b) $1.00075 \times 10^{2} = 100.075$

(c) $3.87 \times 10^{-3} = 0.00387$ (d) $7.0004 \times 10^{7} = 70004000.0$

(e) $0.9813 \times 10^{-5} = 0.000009813$ (f) $3.4567 \times 10^{7} = 34567000$

4.

$$\begin{aligned}
\text{Speed} &= 2.012 \times 10^{3}\ \text{km/h}\\
\text{Time} &= 3\,\text{h}\ 340\ \text{min}\\
\text{Distance} &= \text{Speed} \times \text{Time}\\
&= \left(2.012 \times 10^{3} \times 3\frac{1}{2}\right)\text{km}\\
&= \frac{14084}{2}\ \text{km}\\
&= 7042\,\text{km}\\
&= 7.042 \times 10^{3}\ \text{km}
\end{aligned}$$

5.

$$\begin{aligned}
\text{Speed of light} &= 300000000\ \text{km/h}\\
&= 3.0 \times 10^{8}\ \text{km/h}
\end{aligned}$$

6. $0.000000000000000000000000000001673 = 1.673 \times 10^{30}$ kg

MCQs

1. (b) **2.** (c) **3.** (b) **4.** (b) **5.** (b)

3

Square and Square Roots

Exercise 3.1

1. (a) ✓ (b) ✗ (c) ✗ (d) ✓

(e) ✗ (f) ✓ (g) ✗ (h) ✗

2. (a) The given number = 728 Since, the last digit is 8.

So, 728 is not a perfect square.

(b) The given number = 360

Since, the last digit of '0' of any number in odd number, that number will never be in perfect square.

So, 360 is not a perfect square.

(c) The given number = 1842

Since, the number having 2, 3, 7 or 8 as its units digit can never be a perfect square.

So, 1842 is not a perfect square.

(d) The given number = 49000

Since, the numbers of zeroes at the end of 49000 is odd.

So, 49000 is not a perfect square.

3. (a) $15^2 = 15 \times 15 = 225$

So, last digit is 5.

(b) $24^2 = 24 \times 24 = 576$

so, last digit is 6.

(c) $12^2 = 12 \times 12 = 144$

So, last digit is 4.

(d) $31^2 = 31 \times 31 = 961$

So, last digit is 1.

(e) $38^2 = 38 \times 38 = 1444$

So, last digit is 4.

(f) $43^2 = 43 \times 43 = 1849$

So, last digit is 9.

(g) $51^2 = 51 \times 51 = 2601$

So, last digit is 1.

(h) $23^2 = 23 \times 23 = 529$

So, last digit is 9.

(i) $66^2 = 66 \times 66 = 4356$

So, last digit is 6.

(j) $49^2 = 49 \times 49 = 2401$

So, last digit is 1.

4. (a)

$$\begin{aligned}
1^2 &= 1\\
2^2 &= 1+3\\
3^2 &= 1+3+5\\
4^2 &= 1+3+5+7\\
5^2 &= 1+3+5+7+9\\
6^2 &= 1+3+5+7+9+11\\
7^2 &= 1+3+5+7+9+11+13\\
8^2 &= 1+3+5+7+9+11+13+15\\
9^2 &= 1+3+5+7+9+11+13+15+17\\
10^2 &= 1+3+5+7+9+11+13+15+17+19\\
11^2 &= 1+3+5+7+9+11+13+15+17+19+21\\
12^2 &= 1+3+5+7+9+11+13+15+17+19+21+23
\end{aligned}$$

(b)

$$\begin{aligned}
1^2+2^2+2^2 &= 3^2\\
2^2+3^2+6^2 &= 7^2\\
3^2+4^2+12^2 &= 13^2\\
4^2+\mathbf{5^2}+20^2 &= 21^2\\
5^2+6^2+\mathbf{30^2} &= 31^2\\
6^2+\mathbf{7^2}+\mathbf{42^2} &= \mathbf{43^2}
\end{aligned}$$

(c)

$$\begin{aligned}
11^2 &= 121\\
101^2 &= 10201\\
1001^2 &= 1002001\\
10001^2 &= 100020001\\
100001^2 &= 10000200001\\
1000001^2 &= 1000002000001
\end{aligned}$$

(d)

$$\begin{aligned}
5^2 &= 0\times 100+5^2=25\\
15^2 &= 1\times 200+5^2=225\\
25^2 &= 2\times 300+5^2=625\\
\mathbf{35^2} &= \mathbf{3}\times \mathbf{400}+\mathbf{5^2}=1225\\
\mathbf{45^2} &= 4\times 500+\mathbf{5^2}=2025\\
\mathbf{55^2} &= \mathbf{5}\times \mathbf{600}+\mathbf{5^2}=3025\\
65^2 &= \mathbf{6}\times \mathbf{700}+5^2=\mathbf{4225}\\
\mathbf{85^2} &= \mathbf{8}\times \mathbf{900}+\mathbf{5^2}=\mathbf{7225}
\end{aligned}$$

(e)

$$\begin{aligned}
10^2-10+1 &= 91\\
10^4-10^2+1 &= 9901\\
10^6-10^3+1 &= 999001\\
\mathbf{10^8-10^4+1} &= \mathbf{9990001}\\
\mathbf{10^{10}-10^5+1} &= \mathbf{999900001}\\
\mathbf{10^{12}-10^6+1} &= \mathbf{99999000001}
\end{aligned}$$

Exercise 3.2

1. Subtract 1, 3, 5… and soon, till you get 0.

(a)

$$\begin{aligned}
25-1 &= 24\\
24-3 &= 21\\
21-5 &= 16\\
16-7 &= 9\\
9-9 &= 0
\end{aligned}$$

We subtracted consecutive odd numbers (*i.e.*, 1, 3, 5 …) 5 time.

Thus, $\sqrt{25}=5$

(b)

$$\begin{aligned}
64-1 &= 63\\
63-3 &= 60\\
60-5 &= 55\\
55-7 &= 48\\
48-9 &= 39
\end{aligned}$$

$$\begin{aligned} 39-11 &= 28 \\ 28-13 &= 15 \\ 15-15 &= 0 \end{aligned}$$

We subtracted consecutive odd numbers (*i.e.*, 1, 3, 5 …) 8 times.

Thus, $\sqrt{64}=8$

(c)
$$\begin{aligned} 100-1 &= 99 \\ 99-3 &= 96 \\ 96-5 &= 91 \\ 91-7 &= 84 \\ 84-9 &= 75 \\ 75-11 &= 64 \\ 64-13 &= 51 \\ 51-15 &= 36 \\ 36-17 &= 19 \\ 19-19 &= 0 \end{aligned}$$

We subtracted consecutive odd numbers (*i.e.*, 1, 3, 5, 7 …) 10 times.

Thus, $\sqrt{100}=10$

(d)
$$\begin{aligned} 121-1 &= 120 \\ 120-3 &= 117 \\ 117-5 &= 112 \\ 112-7 &= 105 \\ 105-9 &= 96 \\ 96-11 &= 85 \\ 85-13 &= 72 \\ 72-15 &= 57 \\ 57-17 &= 40 \\ 40-19 &= 21 \\ 21-21 &= 0 \end{aligned}$$

We subtracted consecutive odd numbers (*i.e.*, 1, 3, 5, 7 …) 11 times.

Thus, $\sqrt{121}=11$

2. By prime factorization method :

(a) We have, 576

$$576 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}$$

$$\sqrt{576} = \sqrt{\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}}$$

$$= 2\times 2\times 2\times 3$$

$$= 24$$

Thus, the square root of 576 is 24.

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(b) We have, 1600

$$1600 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{5\times 5}$$

$$\sqrt{1600} = \sqrt{\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{5\times 5}}$$

$$= 2\times 2\times 2\times 5 = 40$$

Thus, the square root of 1600 is 40.

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

(c) We have, 3025

$$3025 = \overline{5\times 5}\times \overline{11\times 11}$$

$$\sqrt{3025} = \sqrt{5\times 5\times 11\times 11}$$

$$= 5\times 11 = 55$$

Thus, the square root of 3025 is 55.

5	3025
5	605
11	121
11	11
	1

(d) We have, 2304

$$2304 = \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{3\times 3}$$

$$\sqrt{2304} = \sqrt{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 3\times 3}$$

$$= 2\times 2\times 2\times 2\times 3$$

$$= 48$$

Thus, the square root of 2304 is 48.

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(e) We have, 7569

$$7569 = \overline{3\times 3}\times \overline{29\times 29}$$

$$\sqrt{7569} = \sqrt{3\times 3\times 29\times 29}$$

$$= 3\times 29$$

$$= 87$$

Thus, the square root of 7569 is 87.

3	7569
3	2523
29	841
29	29
	1

(f) We have, 5184

$$5184 = \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{3\times 3}\times \overline{3\times 3}$$

$$\sqrt{5184} = \sqrt{\overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{3\times 3}\times \overline{3\times 3}}$$

$$= 2\times 2\times 2\times 3\times 3 = 72$$

Thus, the square root of 5184 is 72.

2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

3. By using prime factorization method.

(a) $635 = 5\times 127$

Since, all the prime factors are not in pair.

Thus, the given number is not a perfect square.

5	635
127	127
	1

(b) 1296

$$= \overline{2\times 2}\times \overline{2\times 2}\times \overline{3\times 3}\times \overline{3\times 3}$$

Since, all the prime factors are in pair.

Thus, the given number is a perfect square.

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

(c) $326 = 2 \times 163$

All the prime factors are not in pair.

Thus, the given number is not a perfect square.

2	326
163	163
	1

(d) $6241 = \overline{79 \times 79}$

Since, all the prime factors are in pair.

Thus, the given number is a perfect square.

79	6241
79	79
	1

(e) $11664 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{3 \times 3} \times \overline{3 \times 3} \times \overline{3 \times 3}$

Since, all the prime factors are in pair.

Thus, the given number is a perfect square.

2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(f) $9801 = \overline{3 \times 3} \times \overline{3 \times 3} \times \overline{11 \times 11}$

Since, all the prime factors are in pair.

Thus, the given number is a perfect square.

3	9801
3	3267
3	1089
3	363
11	121
11	11
	1

4. (a) $2700 = \overline{2 \times 2} \times \overline{5 \times 5} \times \overline{3 \times 3} \times 3$

All the prime factors can be paired except 3.

Thus, we should multiply the number by 3.

$2700 \times 3 = (2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5) \times 3$

$8100 = \overline{2 \times 2} \times \overline{5 \times 5} \times \overline{3 \times 3} \times \overline{3 \times 3}$

8100 is a perfect square.

Therefore, the required least number is 3.

And the square root of 8100, *i.e.*,

$\sqrt{8100} = 2 \times 5 \times 3 \times 3 = 90$

2	2700
2	1350
5	675
5	135
3	27
3	9
3	3
	1

(b) $1250 = 2 \times \overline{5 \times 5} \times \overline{5 \times 5}$

All the prime factors can be paired except 2.

Thus, we should multiply the number by 2.

$1250 \times 2 = 2 \times (2 \times 5 \times 5 \times 5 \times 5)$

$2500 = \overline{2 \times 2} \times \overline{5 \times 5} \times \overline{5 \times 5}$

2500 is a perfect square.

Therefore, the required least number is 2.

And the square root of 2500, *i.e.*,

$\sqrt{2500} = 2 \times 5 \times 5 = 50$

2	1250
5	625
5	125
5	25
5	5
	1

(c) $1728 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}\times 3$

All the prime factors can be paired except 3.
Thus, we should multiply the number by 3.

$1728\times 3 = (\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}\times 3)\times 3$

$5184 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}\times\overline{3\times 3}$

5184 is a perfect square.
Therefore, the required least number is 3.
And the square root of 5184, *i.e.*,
$\sqrt{5184} = 2\times 2\times 2\times 3\times 3 = 72$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(d) $343 = \overline{7\times 7}\times 7$

All the prime factors can be paired except 7.
Thus, we should multiply the number by 7.

$343\times 7 = (\overline{7\times 7}\times 7)\times 7$

$2401 = \overline{7\times 7}\times\overline{7\times 7}$

2401 is a perfect square
Therefore, the required least number is 7. And the square root of 2401, *i.e.*,
$\sqrt{2401} = 7\times 7 = 49$

7	343
7	49
7	7
	1

(e) $1568 = \overline{2\times 2}\times\overline{2\times 2}\times 2\times\overline{7\times 7}$

All the prime factors can be paired except 2.
Thus, we should multiply the number by 2.

$1568\times 2 = (\overline{2\times 2}\times\overline{2\times 2}\times 2\times\overline{7\times 7})\times 2$

$3136 = (2\times 2\times 2\times 2\times 2\times 2\times 7\times 7)$

3136 is a perfect square.
Therefore, the required least number is 2.
And the square root of 3136, *i.e.*,
$\sqrt{3136} = 2\times 2\times 2\times 7 = 56$

2	1568
2	784
2	392
2	196
2	98
7	49
7	7
	1

5. The total money collected by two teams = ₹ 338
∴ the money collected by one team = ₹ 338 ÷ 2 = ₹ 169
Since, the contribution of each child are equal to the number of students
So,

$$x\times x = ₹\ 169$$
$$x^2 = ₹\ 169$$
$$x = \sqrt{169}$$
$$x = 13$$

13	169
13	13
	1

Hence, the number of students of each team is 13.

6. The total number of students = 1764
Since, the number of students sitting in each row are equal to the number of rows
So,

$$x\times x = 1764$$
$$x^2 = 1764$$
$$x = \sqrt{1764}$$
$$= \sqrt{2\times 2\times 3\times 3\times 7\times 7}$$
$$x = 2\times 3\times 7 = 42$$

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Hence, 42 rows are needed to make them sit

Exercise 3.3

1. Since, a perfect square has n digits then its square root will have :

1 $\frac{n}{2}$ digits (if n is even)

2. $\left(\frac{n+1}{2}\right)$ digits (if n is odd)

(a) $n = 2$ (even)

So, the number of digits of square root $= \frac{n}{2} = \frac{2}{2} = 1$

(b) $n = 3$ (odd)

So, the number of digits of square root $= \left(\frac{n+1}{2}\right) = \left(\frac{3+1}{2}\right) = 2$

(c) $n = 4$ (even)

So, the number of digits of square root $= \frac{n}{2} = \frac{4}{2} = 2$

(d) $n = 4$ (even)

So, the number of digits of square root $= \frac{n}{2} = \frac{4}{2} = 2$

(e) $n = 5$ (odd)

So, the number of digits of square root $= \left(\frac{n+1}{2}\right) = \left(\frac{5+1}{2}\right) = \frac{6}{2} = 3$

2. By long division method.

(a)

	25
2	6 $\overline{25}$
	−4
45	225
+5	−225
	×

Hence, $\sqrt{625} = 25$

(b)

	27
2	7 $\overline{29}$
	−4
47	3 29
+7	−3 29
	×

Hence, $\sqrt{729} = 27$

(c)

	45
4	$\overline{20}$ $\overline{25}$
	−16
85	425
+5	−425
	×

Hence, $\sqrt{2025} = 45$

(d)

	56
5	$\overline{31}$ $\overline{36}$
	−25
106	636
+6	−636
	×

Hence, $\sqrt{3136} = 56$

(e)

	95
9	$\overline{90}$ $\overline{25}$
	−81
185	925
+5	−925
	×

Hence, $\sqrt{9025} = 95$

3. The remainder is 16. Therefore, 16 should be subtacted from the number 500 to get a perfect square whose square root will be 22.

	22
2	5 $\overline{00}$
	−4
42	100
+2	−84
	16

4. Let us find $\sqrt{1000}$ using long division method. the remainder is 39.

Thus, $31^2 < 1000$

The next perfect square $= 32^2 = 1024$

Hence, the number to be added to make 1000

a perfect square $= 32^2 - 1000 = 1024 - 1000 = 24$

Thus, the required perfect square $= 1000 + 24 = 1024$

Also, $\sqrt{1024} = 32$

	31
3	$\overline{10}\ \overline{00}$
	-9
61	100
$+1$	-61
	39

5. By using prime factorization

$192 = \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times 3$

All the prime factors can be paired except 3.

Thus, we should multiply the number by 3.

When we multiply the number by 3, we get

$192\times 3 = (2\times 2\times 2\times 2\times 2\times 2\times 3)\times 3$

$576 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{3\times 3}$

576 is a perfect square.

Therefore, the required least number is 3.

And the square root of 576, *i.e.,* $\sqrt{576} = 2\times 2\times 2\times 3 = 24$

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

6. First we need to find root of 4000.

This shows that $(63)^2$ is less than 4000 by 31.

So, the number of soldiers left $= 31$

Then, the number of soldiers are arranged in

rows $= 4000 - 31 = 3969$

Let the number of soldiers in each row be x.

Then, number of rows $= x$ $(\because$ both are same)

So, $x\times x = 3969$ $\quad x = \sqrt{3969} = \sqrt{63\times 63} = 63$

Hence, number of soliders is 63 in a row.

	63
6	$\overline{40}\ \overline{00}$
	-36
123	400
$+3$	-369
	31

7. First we need to find root of 1700.

This shows that $(41)^2$ is less than 1700 by 19.

Then, the number of students are arranged in

rows $= 1700 - 19 = 1681$

Let member of students in each row $= x$

Then, numbers of rows $= x$

So, $x\times x = 1681$

$x = \sqrt{1681} = \sqrt{41\times 41} = 41$

Hence, number of rows $= 41$

	41
6	$\overline{17}\ \overline{00}$
	-16
81	100
$+1$	-81
	19

8. We have,

$768 = \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times 3$

Since, the prime factor 3 is not paired, we divide 768 by 3 to make it a perfect square

Let $768 \div 3 = 256 = \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}$

Thus, the required number is 3.

Also, $\sqrt{256} = 2\times 2\times 2\times 2 = 16$

2	768
2	384
2	192
2	96
2	48
2	24

2	12
2	6
3	3
	1

9. First we need to find root of 1024

Let number of trees be x.

Then, the number of rows $= x$

So, $x\times x = 1024$

$x = \sqrt{1024}$

$= \sqrt{32\times 32} = 32$

	32
3	$\overline{10}\ \overline{24}$
	-9
62	124
$+2$	-124
	$\times$

Hence, the number of trees is 32 in a row.

10. The least number divisible by each of the numbers 4, 5 and 10 is determined by finding their LCM.

LCM of 4, 5 and 10 is $2\times 2\times 5 = 20$

Prime factorization of $20 = \underline{2\times 2}\times 5$

The prime factor 5 are not in pairs. This 20 is not a perfect square. In order to make 20 a prefect square, we must pair all the prime factor *i.e.,* 20 must be multiplied by $5 = 5$

Hence, the required smallest perfect square divisible by each of the numebrs 4, 5, 10, is $20\times 5 = 100$

2	4, 5, 10
2	2, 5, 5
5	1, 5, 5
	1, 1, 1

Exercise 3.4

1. (a) $\sqrt{441} = \sqrt{49}\times\sqrt{9}$

LHS $= \sqrt{441}$

$= \sqrt{\overline{3\times 3}\times\overline{7\times 7}}$

$= 3\times 7$

$= 21$

RHS $= \sqrt{49}\times\sqrt{9}$

$= \sqrt{7\times 7}\times\sqrt{3\times 3}$

$= 7\times 3 = 21$

$\therefore$ LHS $=$ RHS

Hence, $\sqrt{441} = \sqrt{49}`\times\sqrt{9}$

3	441
3	147
7	49
7	7
	1

3	9
3	3
	1

7	49
7	7
	1

(b) $\sqrt{441\times 100}` = \sqrt{441}\times\sqrt{100}$

LHS $= \sqrt{441\times 100} = \sqrt{44100}$

$= \sqrt{\overline{2\times 2}\times\overline{3\times 3}\times\overline{5\times 5}\times\overline{7\times 7}}$

$= 2\times 3\times 5\times 7$

$= 210$

RHS $= \sqrt{441}\times\sqrt{100}$

$= \sqrt{3\times 3\times 7\times 7}\times\sqrt{2\times 2\times 5\times 5} = 3\times 7\times 2\times 5 = 210$

$\therefore$ LHS $=$ RHS

Hence, $\sqrt{441\times 100} = \sqrt{441}\times\sqrt{100}$

2	44100
2	22050
3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

(c) $\sqrt{225\times 16} = 60$

LHS $= \sqrt{225\times 16} = \sqrt{3600} = \sqrt{60\times 60} = 60 =$ RHS

$\therefore$ LHS $=$ RHS

Hence, $\sqrt{225\times 16} = 60$

	60
6	$\overline{36}\ \overline{00}$
	-36
120	00
$+0$	-00
	$\times$

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(d) $\sqrt{56\times 9}=\sqrt{144}\times\sqrt{16}$

LHS $=\sqrt{256\times 9}$

$=\sqrt{2304}$

$=\sqrt{\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}}$

$=2\times 2\times 2\times 2\times 3$

$=48$

RHS $=\sqrt{144}\times\sqrt{16}$

$=\sqrt{\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}}\times\sqrt{\overline{2\times 2}\times\overline{2\times 2}}$

$2\times 2\times 3\times 2\times 2\times=48$

∴ LHS $=$ RHS

Hence, $\sqrt{256\times 9}=\sqrt{144}\times\sqrt{16}$

2. (a) $\sqrt{\frac{1}{9}}, \sqrt{1}=\sqrt{1\times 1}=1$

And $\sqrt{9}=\sqrt{3\times 3}=3$

Hence, $\sqrt{\frac{1}{9}}=\frac{1}{3}$

(b) $\sqrt{\frac{25}{49}}$

$\sqrt{25}=\sqrt{5\times 5}=5$

$\sqrt{49}=\sqrt{7\times 7}=7$

Hence, $\sqrt{\frac{25}{49}}=\frac{5}{7}$

(c) $\sqrt{0.04}=\sqrt{\frac{4}{100}}$

$\sqrt{4}=\sqrt{2\times 2}=2$

$\sqrt{100}=\sqrt{10\times 10}=10$

∴ $\sqrt{\frac{4}{100}}=\frac{2}{10}=0.2$

Hence, $\sqrt{0.04}=0.2$

(d) $\sqrt{0.0121}=\sqrt{\frac{121}{10000}}$

$\sqrt{121}=\sqrt{11\times 11}=11$

$\sqrt{10000}=\sqrt{100\times 100}=100$

∴ $\sqrt{\frac{121}{10000}}=\frac{11}{100}=0.11$

Hence, $\sqrt{0.0121}=0.11$

(e) $\sqrt{\frac{324}{784}}$

$\sqrt{324}=\sqrt{2\times 2\times 3\times 3\times 3\times 3}$

$=2\times 3\times 3=18$

$\sqrt{784}=\sqrt{2\times 2\times 2\times 2\times 7\times 7}$

$=2\times 2\times 7=28$

Hence, $\sqrt{\frac{324}{784}}=\frac{18}{28}$

(f) $\sqrt{7.29}=\sqrt{\frac{729}{100}}$

$\sqrt{729}=\sqrt{3\times 3\times 3\times 3\times 3\times 3}$

$=3\times 3\times 3=27$

$\sqrt{100}=\sqrt{10\times 10}=10$

∴ $\sqrt{\frac{729}{100}}=\frac{27}{10}=2.7$

Hence, $\sqrt{7.29}=2.7$

(g) $\sqrt{42.25}=\sqrt{\frac{4225}{100}}$

$\sqrt{4225}=\sqrt{5\times 5\times 13\times 13}$

$=5\times 13=65$

(h) $\sqrt{51.84}=\sqrt{\frac{5184}{100}}$

$$\sqrt{5184}=\sqrt{\begin{matrix}2\times 2\times 2\times 2\times 2\times 2\times 3\\ \times 3\times 3\times 3\end{matrix}}$$

$2\times 2\times 2\times 3\times 3=72$

$$\sqrt{100} = \sqrt{10 \times 10} = 10$$

$$\therefore \quad \sqrt{\frac{4225}{100}} = \frac{65}{10} = 6.5$$

Hence, $\sqrt{42.25} = 6.5$

(i) $\sqrt{50.41} = \sqrt{\frac{5041}{100}}$

$$\sqrt{5041} = \sqrt{71 \times 71} = 71$$

$$\sqrt{100} = \sqrt{10 \times 10} = 10$$

$$\therefore \quad \sqrt{\frac{5041}{100}} = \frac{71}{10} = 7.1$$

Hence, $\sqrt{50.41} = 7.1$

(k) $\sqrt{\frac{529}{625}}$

$$\sqrt{529} = \sqrt{23 \times 23}$$

$$= 23$$

$$\sqrt{625} = \sqrt{25 \times 25}$$

$$= 25$$

Hence, $\sqrt{\frac{529}{625}} = \frac{23}{25}$

$$\sqrt{100} = \sqrt{10 \times 10} = 10$$

$$\therefore \quad \sqrt{\frac{5184}{100}} = \frac{72}{10} = 7.2$$

Hence, $\sqrt{51.84} = 7.2$

(j) $\sqrt{106.09} = \sqrt{\frac{10609}{100}}$

$$\sqrt{10609} = \sqrt{103 \times 103} = 103$$

$$\sqrt{100} = \sqrt{10 \times 10} = 10$$

$$\therefore \quad \sqrt{\frac{10609}{100}} = \frac{103}{10} = 10.3$$

Hence, $\sqrt{106.09} = 10.3$

(l) $\sqrt{1\frac{63}{81}} = \sqrt{\frac{144}{81}}$

$$= \sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$$

$$= 2 \times 2 \times 3 = 12$$

$$\sqrt{81} = \sqrt{3 \times 3 \times 3 \times 3}$$

$$= 3 \times 3 = 9$$

Hence, $\sqrt{1\frac{63}{81}} = \frac{12}{9}$

(m) $\sqrt{\frac{0.0441}{0.0256}} = \sqrt{\frac{441}{256}}$

$$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7} = 3 \times 7 = 21$$

$$\sqrt{256} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2 \times 2 \times 2 \times 2 = 16$$

$$\therefore \quad \sqrt{\frac{441}{256}} = \frac{21}{16}$$

Hence, $\sqrt{\frac{0.0441}{0.0256}} = \frac{0.21}{0.16}$

3. (a) $\frac{\sqrt{13.69} + \sqrt{18.49}}{\sqrt{151.29} - \sqrt{5.29}} = \frac{3.7 + 4.3}{12.3 - 2.3} = \frac{8}{10} = \frac{4}{5} = 0.8$

(b) $\sqrt{\frac{7\frac{1}{5}}{5}} + \sqrt{\frac{3\frac{1}{8}}{8}} - \sqrt{\frac{8\frac{1}{10}}{10}} = \sqrt{\frac{\frac{36}{5}}{5}} + \sqrt{\frac{\frac{25}{8}}{8}} - \sqrt{\frac{\frac{81}{10}}{10}}$

$$= \sqrt{\frac{36}{25}} + \sqrt{\frac{25}{64}} - \sqrt{\frac{81}{100}}1$$

$$= \sqrt{\frac{6 \times 6}{5 \times 5}} + \sqrt{\frac{5 \times 5}{8 \times 8}} - \sqrt{\frac{9 \times 9}{10 \times 10}}$$

$$= \frac{6}{5} + \frac{5}{8} - \frac{9}{10}$$

$$= \frac{6 \times 8 + 5 \times 5 - 9 \times 4}{40} = \frac{48 + 25 - 36}{40} = \frac{37}{40}$$

4. (a) We have,

	1.732
1	$3.\overline{00}\ \overline{00}\ \overline{00}$
	-1
27	200
$+7$	-189
343	1100
$+3$	-1029
3462	7100
$+2$	-6924
	176 R

Hence, $\sqrt{3} = 1.73$

(b) We have,

	2.236
2	$5.\overline{00}\ \overline{00}\ \overline{00}$
	-4
42	100
$+2$	-84
443	1600
$+3$	-1329
4466	27100
$+6$	-26796
	304 R

Hence, $\sqrt{5} = 2.23$

(c) We have, $\frac{4}{7} = 0.5714$

	0.755
7	$0.\overline{57}\ \overline{14}\ \overline{28}$
	-49
145	814
$+5$	-725
1505	8928
$+5$	-7525
	1403 R

Hence, $\sqrt{\frac{4}{7}} = \sqrt{0.5714} = 0.75$

(d) We have, $\frac{13}{11} = 1.1818181818$

	1.0871
1	$1.\overline{18}\ \overline{18}\ \overline{18}\ \overline{18}$
	-1
208	1818
$+8$	-1664
2161	15418
$+7$	-15169
21741	24918
$+1$	-21741
	3177 R

Hence, $\sqrt{\frac{13}{11}} = = \sqrt{1.18181818}$

(e) We have $1\frac{5}{9} = \frac{14}{9} = 1.5555$

	1.247
1	$1.\overline{55}\ \overline{15}$
	-1
22	55
$+2$	-44
244	1155
$+4$	-976
	179 R

Hence, $\sqrt{1\frac{5}{9}} = 1.24$

(f) We have, $83\frac{7}{11} = \frac{920}{11} = 83.63$

	9.145
9	$\overline{93}.\overline{63}\ \overline{63}$
	-81
181	263
$+1$	-181
1824	8263
	-7296
	967 R

Hence, $\sqrt{83\frac{7}{11}} = \sqrt{\frac{920}{11}} = 9.14$

(g) We have, $619\frac{2}{5}=\frac{3097}{5}=619.40$

	24.887
2	$\overline{6}\,\overline{19}.\overline{40}\,\overline{00}\,\overline{00}$
	−4
44	219
+4	−176
488	4340
+8	−3904
4968	43600
+8	−39744
49767	385600
+7	−348369
	37321 R

Hence, $\sqrt{619\frac{2}{5}}=24.88$

5. Let the number which when multiplied by itself be *x*.

Then,
$$\begin{aligned} x\times x &= 944.57 \\ x &= \sqrt{944.57} \\ x &= \sqrt{30.733\times 30.733} \\ x &= 30.733 \end{aligned}$$

Hence the number is 30.733

	30.733
3	$\overline{9}\,\overline{44}.\overline{57}\,\overline{00}\,\overline{00}$
	−9
607	4457
+7	−4249
6143	20800
+3	−18429
61463	237100
+3	−184389
	52711 R

MCQs

1. (a) **2.** (d) **3.** (b) **4.** (b) **5.** (b) **6.** (c)

4 Cube and Cube Roots

Exercise 4.1

1. (a) $8^3=8\times 8\times 8=512$

(b) $11^3=11\times 11\times 11=1331$

(c) $14^3=14\times 14\times 14=2744$

(d) $19^3=19\times 19\times 19=6859$

(e) $0.5^3=0.5\times 0.5\times 0.5=0.125$

(f) $35^3=35\times 35\times 35=42875$

(g) $(-4)^3=(-4)\times(-4)\times(-4)=16\times(-4)=-64$

(h) $\left(\frac{-13}{8}\right)^3=\left(\frac{-13}{8}\right)\times\left(\frac{-13}{8}\right)\times\left(\frac{-13}{8}\right)$

$$=\left(\frac{169}{64}\right)\times\left(\frac{-13}{8}\right)=\frac{2197}{512}$$

(i) $1.8^3 = 1.8\times 1.8\times 1.8 = 3.24\times 1.8 = 5.832$

(j) $2.2^3 = 2.2\times 2.2\times 2.2 = 484\times 2.2 = 10.648$

(k) $(0.09)^3 = 0.09\times 0.09\times 0.09 = 0.0081\times 0.09 = 0.000729$

(l) $\left(3\frac{1}{5}\right)^3 = \left(\frac{16}{5}\right)^3 = \frac{16}{5}\times\frac{16}{5}\times\frac{16}{5} = \frac{256}{25}\times\frac{16}{5} = \frac{4096}{125}$

2. (a) Resolving 3575 into prime factors, we get

Thus, $3575 = 5\times 5\times 11\times 13$

Here, all prime factors are ungrouped

Hence, 3575 is not a perfect cube.

5	3575
5	715
11	143
13	13
	1

(b) Resolving 1728 into prime factors, we get,

Thus, $1728 = \overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{3\times 3\times 3}$

Since, 1728 can be grouped into triplet of equal factors.

Hence, it is a perfect cube.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
1	1

(c) Resolving 27000 into prime factors, we get

Thus, $27000 = \overline{2\times 2\times 2}\times\overline{3\times 3\times 3}\times\overline{5\times 5\times 5}$

Since, 27000 can be grouped into triplet of equal factors.

Hence, it is a perfect cube.

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

(d) Resolving 9261 into prime factors, we get

Thus, $9261 = \overline{3\times 3\times 3}\times\overline{7\times 7\times 7}$

Since 9261 can be grouped in triplet of equal factors.

Hence, it is a perfect cube.

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

3. We have, 392 the prime factors of

$392 = \overline{2\times 2\times 2}\times 7\times 7$

Since, the prime factor 7 does not appear in a group of three. Hence, 392 is not a perfect cube.

One more factor 7 is required to make triplets of 7.

If we multiply the number by 7, 392 will become a perfect cube.

Hence, the required number is 7.

2	392
2	196
2	98
7	49
7	7
	1

4. We have, 17496

The prime factors of

$17496 = \overline{2\times 2\times 2}\times \overline{3\times 3\times 3}\times \overline{3\times 3\times 3}\times 3$

Since, the prime factor 3 does not appear in a group of three, hence, 17496 is not a perfect cube.

Two more factor 3×3 is required to make triplets. If we multiply the number by 3×3, 17496 will become a perfect cube.

Hence, the required number is 9.

2	17496
2	8748
2	4374
3	2187
3	729
3	243

3	81
3	27
3	9
3	3
	1

5. Again on factorizing 8640, we get,

$8640 = \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}\times \overline{3\times 3\times 3}\times 5$

Since, the prime factor 5 does not appear in a group of three's, we must remove the one extra factors *i.e.*, 5 so as to make the number a perfect cube.

Thus, if we divide the given number 8640 by 5, the resulting number has prime factors in a group of three's.

i.e., $8640 \div 5 = 1728 = \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}\times \overline{3\times 3\times 3}$

Hence, the required number is 5.

2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

6. Again on factorizing 46305, we get,

$46305 = \overline{3\times 3\times 3}\times 5\times \overline{7\times 7\times 7}$

Since, the prime factor 5 does not appear in a group of three's we must remove the one extra factors *i.e.*, 5 so as to make the number a perfect cube.

Thus, if we divide the given numbers 46305 by 5, the resulting number has prime factors in a group of three's

i.e., $46305 \div 5 = 9261$

$= \overline{3\times 3\times 3}\times \overline{7\times 7\times 7}$

Hence, the required number is 5.

3	46305
3	15435
3	5145
5	1715
7	343
7	49
7	7
	1

7. The side of a cube $= 6.5$ cm

Since, the volume of a cube $= \text{side}^3 = (6.5)^3 \text{ cm}^3 = 6.5\times 6.5\times 6.5 \text{ cm}^3$

$= 42.25 \times 6.5 \text{ cm}^3$

$= 274.625 \text{ cm}^3$

Hence, the volume of a cube is 274.625 cm^3

8. (a) 343 Since, the last digit of 343 is odd *i.e.*, 3.

So, the last digit of (343) is also odd.

(b) 1331 Since, the last digit of 1331 is odd, *i.e.*, 1

So, the last digit of (1331) is also odd.

(e) 512 Since, the last digit of 512 is even, *i.e.*, 2.

So, the last digit of (512) is also even.

9. (a) and (d) are the cubes of negative integers.

10. (a) True (b) False (c) True (d) False (e) True

Exercise 4.2

1. Subtract 1, 7, 19, 37 … and soon successively from the number, till you get 0.

(a) we have,

$64-1=63$

$63-7=56$

$56-19=37$

$37-37=0$

We subtracted number (1, 7, 19 …) 4 times.

Thus, $\sqrt[3]{64}=4$

(b) We have, $343-1=342$

$342-7=335$

$335-19=316$

$316-37=279$

$279-61=218$

$218-91=127$

$127-127=0$

We subtracted number (1, 7, 19 …,) 7 times.

Thus, $\sqrt[3]{343}=7$

(c) We have,

$512-1=511$

$511-7=504$

$504-19=485$

$485-37=448$

$448-61=387$

$387-91=296$

$296-127=169$

$169-169=0$

we subtracted numbers (1, 7, 19 …,) 8 times.

Thus, $\sqrt[3]{512}=8$

(d) We have,

$729-1=728$

$728-7=721$

$721-19=702$

$702-37=665$

$665-61=604$

$604-91=513$

$513-127=386$

$386-169=217$

$217-217=0$

We subtracted number 1, 7, 19, 37 …, 9 times

Thus, $\sqrt[3]{729}=9.$

2. By using prime factorization method.

(a) We have, $2744=\overline{2\times2\times2}\times\overline{7\times7\times7}$

Thus, $\sqrt[3]{2744}=\sqrt[3]{\overline{2\times2\times2}\times\overline{7\times7\times7}}$

$\sqrt[3]{2744}=2\times7=14$

Hence, the cube root of 2744 is 14.

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

(b) We have,

$15625=\overline{5\times5\times5}\times\overline{5\times5\times5}$

Thus, $\sqrt[3]{15625}=\sqrt[3]{\overline{5\times5\times5}\times\overline{5\times5\times5}}$

$\sqrt[3]{15625}=5\times5=25$

Hence, the cube root of 15625 is 25.

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

(c) We have,

$-10648=\overline{(-2)\times(-2)\times(-2)}\times\overline{11\times11}$

Thus, $=-1\sqrt[3]{\overline{2\times2\times2}\times\overline{11\times11\times11}}$

$=-2\times11=-22$

Hence, the cube root of -10648 is -22..

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

(d) We have, $13824 = \overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{3\times 3\times 3}$

Thus, $\sqrt[3]{13824} = \sqrt[3]{\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{3\times 3\times 3}}$

$\sqrt[3]{13824} = 2\times 2\times 2\times 3 = 24$

Hence, the cube root of 13824 is 24.

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(e) We have,
$474552 = \overline{2\times 2\times 2}\times\overline{3\times 3\times 3}\times\overline{13\times 13\times 13}$

Thus, $\sqrt[3]{474552}$

$= \sqrt[3]{2\times 2\times 2\times 3\times 3\times 3\times 13\times 13\times 13}$

$= 2\times 3\times 13 = 78$

Hence, the cube roots of 474552 is 78.

2	474552
2	237276
2	118638
3	59319
3	19773
3	6591
13	2197
13	169
13	13
	1

(f) We have, $\dfrac{125}{3375} = \dfrac{\overline{5\times 5\times 5}}{\overline{3\times 3\times 3}\times\overline{5\times 5\times 5}}$

Thus, $= \dfrac{1}{\overline{3\times 3\times 3}}$

$\sqrt[3]{\dfrac{125}{3375}} = \sqrt[3]{\dfrac{1}{3\times 3\times 3}} = \dfrac{1}{3}$

Hence, the cube roots of $\sqrt{\dfrac{125}{3375}}$ is $\dfrac{1}{3}$.

5	125
5	25
5	5
	1

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

(g) We have, $\dfrac{-1000}{1331} = \dfrac{\overline{(-10)\times(-10)\times(-10)}}{\overline{11\times 11\times 11}}$

Thus, $\sqrt[3]{\dfrac{-1000}{1331}} = \sqrt{\dfrac{\overline{(-10)\times(-10)\times(-10)}}{\overline{11\times 11\times 11}}}$

$= \dfrac{-10}{11}$

Hence, the cube roots of $\sqrt{\dfrac{-1000}{1331}}$ is $\dfrac{-10}{11}$

10	1000
10	100
10	10
	1

10	1331
10	121
10	11
	1

(h) We have,

$$\frac{-4096}{-9261}=\frac{4096}{9261}$$

$$=\frac{\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}}{\overline{3\times 3\times 3}\times\overline{7\times 7\times 7}}$$

Thus, $\sqrt[3]{\frac{4096}{9261}}$

$$=\sqrt[3]{\frac{\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}}{\overline{3\times 3\times 3}\times\overline{7\times 7\times 7}}}$$

Hence, the cube roots of $\sqrt{\frac{4096}{9261}}$ is $\frac{-16}{-21}$.

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

3. By using prime factorization method.

(a) $6859=\overline{19\times 19\times 19}$

$\sqrt[3]{6859}=\sqrt{19\times 19\times 19}$

$=19$

Hence, the cube root of 6859 is 19.

19	6859
19	361
19	19
	1

(b) $15625=\overline{5\times 5\times 5}\times\overline{5\times 5\times 5}$

$\sqrt[3]{15625}=\sqrt{\overline{5\times 5\times 5}\times\overline{5\times 5\times 5}}$

$=5\times 5=25$

Hence, the cube root of 15625 is 25.

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

(c) $-1728=-1\times\overline{2\times 2\times 2}\times\overline{2\times 2\times 2}\times\overline{3\times 3\times 3}$

$-\sqrt{1728}=-\sqrt{2\times 2\times 2\times 2\times 2\times 3\times 3\times 3}$

$=-(2\times 2\times 3)=-12$

Hence, the cube root of (-1728) is -12.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(d) $74088=\overline{2\times 2\times 2}\times\overline{3\times 3\times 3}\times\overline{7\times 7\times 7}$

$\sqrt[3]{74088}=\sqrt{\overline{2\times 2\times 2}\times\overline{3\times 3\times 3}\times\overline{7\times 7\times 7}}$

$=2\times 3\times 7$

$=42$

Hence, the cube root of 74088 is 42.

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
3	343
7	49
7	7
	1

4. (a) $\sqrt[3]{8\times 64}=8$

LHS $=\sqrt[3]{8\times 64}$

$=\sqrt[3]{8\times 8\times 8}$

$=8=\text{RHS}$

Hence, $\sqrt[3]{8\times 64}=8$

(b) $\sqrt[3]{64}\times\sqrt[3]{1331}=\sqrt[3]{85184}$

LHS $=\sqrt[3]{4\times 4\times 4}\times\sqrt[3]{11\times 11\times 11}$

$=4\times 11=44$

RHS $=\sqrt[3]{4\times 4\times 4\times 11\times 11\times 11}$

$=4\times 11=44$

Hence, $\sqrt[3]{64}\times\sqrt[3]{1331}=\sqrt[3]{85184}$

(c) $\dfrac{\sqrt[3]{729}}{\sqrt[3]{216}}=\sqrt[3]{3.375}$

LHS $=\sqrt[3]{\dfrac{3\times 3\times 3\times 3\times 3\times 3}{2\times 2\times 2\times 3\times 3\times 3}}$

$\dfrac{3}{2}=1.5$

$=1.5\times 1.5\times 1.5$

$=(1.5)^3$

$=\sqrt[3]{3.375}$

$=\text{RHS}$

Hence, $\dfrac{\sqrt[3]{729}}{\sqrt[3]{216}}=\sqrt[3]{3.375}$

(d) $\sqrt[3]{27}\times\sqrt[3]{343}=\sqrt[3]{9261}$

LHS $=\sqrt[3]{27}\times\sqrt[3]{343}$

$=\sqrt[3]{3\times 3\times 3}\times\sqrt[3]{7\times 7\times 7}$

$=3\times 7$

$=21$

RHS $=\sqrt[3]{9261}$

$=\sqrt{3\times 3\times 3\times 7\times 7\times 7}$

$=3\times 7$

$=21$

$\therefore$ LHS $=$ RHS

$\therefore$ Hence, $\sqrt[3]{27}\times\sqrt[3]{343}=\sqrt[3]{9261}$

5. (a) $243=3\times 3\times 3\times 3\times 3$

Since, the prime factor 3 does not appear in a group of three, one more factor 3 is required to make triplets of 3. If we multiply the number by 3, 243 will become a perfect cube.

Hence, the required number is 3.

(b) $2916=2\times 2\times 3\times 3\times 3\times 3\times 3\times 3$

Since, the prime factor 2 does not appear in a group of three, one more factor 2 is required to make triplets of 2.

It we multiply the number by 2, 2916 will become a perfect cube.

Hence, the required number is 2.

(c) $1372=2\times 2\times 7\times 7\times 7$

Since, the prime factor 2 does not appear in a group of three, one more factor 2 is required to make triplets of 2. It we multiply the number by 2, 1372 will become a perfect cube.

Hence, the required number is 2.

(d) $11664=\overline{2\times 2\times 2}\times 2\times\overline{3\times 3\times 3}\times\overline{3\times 3\times 3}$

Since, the prime factor 2 does not appear in required to make triplets of 2. If we multiply the number by 4. 11664 will become a perfect cube.

Hence, the required number is 4.

6. (a) $15625 = \overline{5\times 5\times 5}\times \overline{5\times 5\times 5}$

Thus, $\sqrt[3]{15625} = \sqrt[3]{5\times 5\times 5\times 5\times 5\times 5}$

$= 5\times 5$

$= 25$

Hence, the ones digit is 5.

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

(b) $68921 = \overline{41\times 41\times 41}$

Thus, $\sqrt[3]{68921} = \sqrt[3]{41\times 41\times 41} = 41$

Hence, the ones digit is 1.

41	68921
41	1681
41	41
	1

(c) $32768 = \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}$

Thus,

$\sqrt[3]{32768} = \sqrt{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2}$

$= 2\times 2\times 2\times 2\times 2 = 32$

Hence, the ones digit is 2.

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(d) $117649 = \overline{7\times 7\times 7}\times \overline{7\times 7\times 7}$

Thus, $\sqrt[3]{117649} = \sqrt{7\times 7\times 7\times 7\times 7\times 7}$

$= 7\times 7 = 49$

Hence, the ones digit is 49.

7	117649
7	16807
7	2401
7	343
7	49
7	7
	1

7. (a) The ones digit of 12167 is 7. Therefore, the ones digit of the cube root is 3. After ignoring the last three digit (1, 6, 7), we get 12. The cube root of 8 is 2. Therefore, the tens digit of the cube root of 12167 is 2. Hence, the cube root of 12167 is 23.

(b) The ones digit of 2197 is 7. Therefore, the ones digit of the cube root is 3. After ignoring the last three digit (1, 9, 7), we get 2. The cube root of 1 is 1. Therefore, the tens digits of the cube root of 2197 is 1. Hence, the cube root of 2197 is 13.

(c) The ones digit of 15625 is 5. Therefore, the ones digit of the cube root is 5. After ignoring the last three digit (6, 2 and 5), we get 15. The cube root of 8 is 2.

Therefore, the tens digit of the cube of 15625 is 2. Hence, the cube root of 15625 is 25.

(d) The ones digit of 74088 is 8. Therefore, the ones digit of the cube root is 2. After ignoring the last three digit (0, 8, 8), we get 74. The cube root of 64 is 4.

Therefore, the tens digit of the cube root of 74088 is 4. Hence, the cube root of 74088 is 42.

8. (a) Again on factorizing 250, we get,

$250 = 2\times 5\times 5\times 5$

Since, the prime factor 2 does not appear in a group of three, we must remove the one extra factor, so as to make the number a perfect cube.

Thus, if we divide the given number 250 by 2 the resulting number has prime factors in a group of three.

$i.e., 250 \div 2 = 125 = \overline{5\times 5\times 5}$

Hence, the required number is 2.

(b) On factorizing 1715, we get,

$1715 = 5\times \overline{7\times 7\times 7}$

Since, the prime factor 5 does not appear in a group of three, we must remove the one extra factor so as to make the number a perfect cube.

Thus, if we divide the given number 1715 by 5 the resulting number has prime factors in a group of three.

$i.e., 1715 \div 5 = 343 = \overline{7\times 7\times 7}$

Hence, the required number is 5.

(c) On factorizing 16875, we get

$16875 = \overline{5\times 5\times 5}\times 5\times \overline{3\times 3\times 3}$

Since, the prime factor 5 does not appear in a group of three, we must remove the one extra factor so as to make the number a perfect cube

Thus, if we divide the given number 16875 by 5 the resulting number has prime factors in a group of three.

$i.e., 16875 \div 5 = 3375 = \overline{3\times 3\times 3}\times \overline{5\times 5\times 5}$

Hence, the required number is 5.

(d) On factorizing 65856, we get

$65856 = \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}\times 3\times \overline{7\times 7\times 7}$

Since, the prime factor 3 does not appear in a group of three, we must remove the one extra factor so as to make the number a perfect cube.

Thus, if we divide the given number 65856 by 3, the resulting number has prime factors in a group of three.

$i.e., 65856 \div 3 = 21952 = \overline{2\times 2\times 2}\times \overline{2\times 2\times 2}\times \overline{7\times 7\times 7}$

Hence, the required number is 3.

MCQs

1. (c) **2.** (b) **3.** (b) **4.** (b) **5.** (b)

5 Direct and Inverse Variation

Exercise 5.1

1. (a) First calculate the ratio of *a* and *b*.

$$\frac{a}{b}=\frac{1}{4}=\frac{4}{16}=\frac{6}{24}=\frac{5}{20}=\frac{9}{36}=\frac{1}{4}$$

We can see that though the value of *b* increases with the increase in the value of *a*, the ratio of *a* and *b* is constant, *i.e.*, 1 : 4. Thus, *a* and *b* vary directly and the constant of variation is $\frac{1}{4}$.

(b) First calculate the ratio of *a* and *b*.

$$\frac{a}{b}=\frac{4}{8}\neq\frac{5}{6}\neq\frac{7}{14}\neq\frac{10}{6}\neq\frac{6}{5}$$

So, *a* and *b* are not vary directly.

(c) First calculate the ratio of *a* and *b*.

$$\frac{a}{b}=\frac{1}{22}\neq\frac{2}{33}\neq\frac{3}{44}\neq\frac{4}{55}\neq\frac{5}{66}$$

So, *a* and *b* are not vary directly.

(d) First calculate the ratio of *a* and *b*.

$$\frac{a}{b}=\frac{3^2}{3^3}=\frac{3^3}{3^4}=\frac{3^4}{3^5}=\frac{3^5}{3^6}=\frac{3^6}{3^7}=\frac{1}{3}$$

We can see that though the value of *b* increases with the increase in the value of *a*, the ratio of *a* and *b* is constant, *i.e.*, 1 : 3. Thus, *a* and *b* vary directly and the constant of variation is $\frac{1}{3}$.

(e) First calculate the ratio of *a* and *b*.

$$\frac{a}{b}=\frac{4}{10}=\frac{6}{15}=\frac{10}{25}=\frac{16}{40}=\frac{20}{50}=\frac{2}{5}$$

We can see that though the value of *b* increases with the increase in the value of *a*, the ratio of *a* and *b* is constant, *i.e.*, 2 : 5. Thus, *a* and *b* vary directly and the constant of variation is $\frac{2}{5}$.

(f) First calculate the ratio of *a* and *b*.

$$\frac{a}{b}=\frac{4}{9}\neq\frac{6}{13.5}\neq\frac{10}{22.5}\neq\frac{16}{36}\neq\frac{28}{63}$$

So, *a* and *b* are not vary directly.

2. Since, *p* and *q* vary directly.

(a) So, 4 : 10 :: 6 : ?

$$\frac{4}{10}=\frac{6}{?} \qquad ?=\frac{6\times 10}{4}=3\times 5=15$$

Similarly,

$$?:20::21:30$$

$$\frac{?}{20}=\frac{21}{30}$$

$$? = \frac{20 \times 21}{30} = 2 \times 7 = 14$$

So, the table are :

p	4	10	**14**	20
q	6	**15**	21	30

(b) So, 2 : 6 :: 5 : ?

$$\frac{2}{6} = \frac{5}{?}$$

$$? = \frac{5 \times 6}{2}$$

$$? = 5 \times 3 = 15$$

Similarly,

? : 17.5 :: 8 : 20

$$\frac{?}{17.5} = \frac{8}{20}$$

$$? = \frac{8 \times 17.5}{20}$$

$$? = \frac{140}{20} = 7$$

So, the table are :

p	2	6	**7**	8
q	5	**15**	17.5	20

(c) So, 3.6 : 5 :: ? : 10

or

$$\frac{3.6}{5} = \frac{?}{10}$$

$$? = \frac{3.6 \times 10}{5}$$

$$? = 3.6 \times 2 = 7.2$$

Similarly, 5 : 9.5 :: 10 : ?

$$\frac{5}{9.5} = \frac{10}{?}$$

$$? = \frac{10 \times 9.5}{5}$$

$$? = 2 \times 9.5$$

$$? = 19$$

9.5 : 17.1 :: 19 : ?

$$\frac{9.5}{17.1} = \frac{19}{?}$$

$$? = \frac{19 \times 17.1}{9.5}$$

$$? = \frac{324.9}{9.5}$$

$$? = 34.2$$

And 17.1 : ? :: 34.2 : 11

$$\frac{17.1}{?} = \frac{34.2}{11}$$

$$? = \frac{17.1 \times 11}{34.2}$$

$$? = \frac{188.1}{34.2}$$

$$? = 5.5$$

So, the table are :

p	3.6	5	9.5	17.1	5.5
q	**7.2**	10	**19**	**34.2**	11

3. (a), (b) and (d) are vary directly with each other.

4. The cost of 11 pens = ₹ 55

∴ the cost of 1 pen = ₹ 55 ÷ 11 = ₹ 5

∴ the cost of 4 pens = ₹ 5 × 4 = ₹ 20

∴ the cost of 5 pens = ₹5 × 5 = ₹ 25

∴ the cost of 6 pens = ₹ 5 × 6 = ₹ 30

∴ the cost of 7 pens = ₹5 × 7 = ₹ 35

∴ the cost of 8 pens = ₹ 5 × 8 = ₹ 40

5. The cost of 10 metres of cloth = ₹ 250

∴ the cost of 1 metre of cloth = ₹ 250 ÷ 10 = ₹ 25

∴ the cost of 5.8 metre of cloth = ₹ 25 × 5.8 = ₹ 145

6. The cost of 2 dozens or 24 bananas = ₹ 72

∴ cost of 1 banana = ₹ 72 ÷ 24 = ₹ 3

∴ the cost of 105 bananas = ₹ 3 × 105 = ₹ 315

7. Since, the given two quantities vary directly.

mass (g)	180	105
length (cm)	15	x

Therefore, ratio of mass = ratio of their length, $180 : 105 = 15 : x$

$$\frac{180}{105} = \frac{15}{x} \quad \Rightarrow \quad x = \frac{15 \times 105}{180} = \frac{105}{12} = 8.75 \text{ cm}$$

Hence, 8.75 cm long of the rod whose mass is 105 g.

8. As the weight of food increases the number of persons also increase, this is a case of direct variation.

Let the required food for 25 persons be x kg.

Food (kg)	85	x
No. of persons	5	25

Now, $85 : x = 5 : 25$

$$\frac{85}{x} = \frac{5}{25}$$

$$x = \frac{85 \times 25}{5} \text{ kg}$$

$$x = (85 \times 5)\,\text{kg} = 425\,\text{kg}$$

Thus, the required food is 425 kg.

9. Let the amount of milk be x litres.

So,

Milk (l)	10	x
Water (l)	2	15

Now,

$$10 : x = 2 : 15$$
$$\frac{10}{x} = \frac{2}{15}$$
$$x = \frac{10 \times 15}{2}\text{ litres}$$
$$x = 5 \times 15\text{ litres}$$

Hence, 75 l is the amount of milk required.

10. Since, words are increase then time will also increase.

So, let the required time be x min.

Type word	1920	2304
Time	60	x

Now,

$$1920 : 2304 = 60 : x$$
$$\frac{1920}{2304} = \frac{60}{x}$$
$$x = \frac{60 \times 2304}{1920}\text{ min}$$
$$x = \frac{2304}{32}\text{ min} = 72\text{ min}$$

Hence, the required time is 72 minutes.

11. Let he paid for 17 packs of paper be ₹ x.

So,

Cost (₹)	112	x
Packs of paper	7	17

Now,

$$112 : x = 7 : 17$$
$$\frac{112}{x} = \frac{7}{17}$$
$$x = ₹\,\frac{112 \times 17}{7}$$
$$x = ₹\ 16 \times 17 = ₹\ 272$$

Hence, he will pay ₹ 272 for 17 packs of paper.

12. Let the interest after 3 years be ₹ x.

Then, we write the information in the table as shown below :

Deposit (₹)	3000	3600
Interest (₹)	750	x

Now, 3000 : 3600 :: 750 : x

$$\frac{3000}{3600} = \frac{750}{x}$$

$$x = ₹\,\frac{3600 \times 750}{3000}$$

$$x = ₹\,\frac{36 \times 75}{3}$$

$$x = ₹\,36 \times 25 = ₹\,900$$

Hence, ₹ 900 will be gotten after 3 years as interest on deposit of ₹ 3600.

13. Let the required number of boxes box.

Then, we write the information in the table as shown below :

No. of Boxes	68	x
Length of shelf (cm)	13.6	20.4

Now,

$$68 : x = 13.6 : 20.4$$

$$\frac{68}{x} = \frac{13.6}{20.4}$$

$$x = \frac{68 \times 20.4}{13.6}$$

$$x = 5 \times 20.4$$

$$x = 102 \text{ boxes}$$

Hence, the number of boxes is 102.

14. Let the required quantity of cereals be x kg for 165 members during the month,

Then, we write the information in the table as shown below :

Cereals (kg)	1530	x
No. of member	102	165

Now,

$$1530 : x = 102 : 165$$

$$\frac{1530}{x} = \frac{102}{165}$$

$$x = \frac{1530 \times 165}{102} \text{ kg}$$

$$= 15 \times 165 \text{ kg} = 2475 \text{ kg}$$

Hence, the quantity of cereals is 2475 kg for 165 members during the month.

15. Let the required number of men be x for digging 225 metre long trench.

Then, we write the information in the table as shown below :

Men	15	x
Trench (m)	135	225

Now,

$$15 : x = 135 : 225$$

$$\frac{15}{x} = \frac{135}{225}$$

$$x = \frac{15 \times 225}{135}$$

$$= \frac{225}{9} = 25 \text{ men}$$

Hence, the required number of men is 25 for digging 225 metre long trench in a day.

Exercise 5.2

1. (a) Here, $p \times q = 4 \times 9 = 3 \times 12 \neq 6 \times 8 \neq 1 \times 36.$

Therefore, p and q are not vary inversely.

(b) Here, $p \times q = 9 \times 5 = 10 \times 4.5 = 12 \times 3.75 = 15 \times 3 = 45.$

Therefore, p and q vary inversely with each other.

(c) Here, $p \times q = 24 \times 4 = 12 \times 8 = 16 \times 6 \neq 48 \times 12 \neq 192 \times 0.5$

Therefore, p and q are not vary inversely.

(d) Here, $p \times q = 4 \times 7 \neq 2 \times 4.5 \neq 3 \times 1.5 \neq 3 \times 11 \neq 11 \times 12$

Therefore, p and q are not vary inversely.

2. Since, volume (V) and pressure (P) vary inversely.

Therefore,

$$48 : 60 = \frac{3}{2} : b$$

$$\frac{48}{60} = \frac{\frac{3}{2}}{b}$$

$$b = \frac{60 \times 3}{48 \times 2}$$

$$b = \frac{30}{16} = 1.875$$

Also,

$$a : 48 = b : 2$$

$$\frac{a}{48} = \frac{b}{2}$$

$$a = \frac{48 \times b}{2}$$

$$a = \frac{48 \times 1.875}{2} \quad (\because b = 1.875)$$

$$a = 45$$

Also,

$$60 : c = 1 : \frac{3}{2}$$

$$\frac{60}{c} = \frac{1}{\frac{3}{2}}$$

$$c = \frac{60 \times 3}{2} = 30 \times 3 = 90$$

And,

$$c : 100 = d : 1$$

$$\frac{c}{100} = \frac{d}{1}$$

$$d = \frac{c}{100}$$

$$= \frac{90}{100} \quad (\because c = 90)$$

$$d = 0.9$$

Thus, complete table is given below :

Volume (in cm^3)	**45**	48	60	**90**	100
Pressure (in atmosphere)	2	**1.875**	3\2	1	**0.9**

3. The total number of pencils $= 80$

(a) $\therefore$ each students get the pencils $= 80 \div 5 = 16$ pencils

(b) $\therefore$ each students get the pencils $= 80 \div 10 = 8$ pencils

4. More men, less days, so it is a problem of inverse variation. We have the following table :

Number of men	40	50
Number of days	10	x

Now, $40 : 50 :: x : 10$

$$\frac{40}{50} = \frac{x}{10}$$
$$x = \frac{40 \times 10}{50} \text{ days}$$
$$x = 8 \text{ days}$$

5. For 500 men, foot last 8 week.

Suppose for 400 men, food last x weeks.

Less men, more weeks, so it is a problem of inverse variation. We have the following table :

No. of men	500	400
No. of weeks	8	x

Now,
$$500 : 400 = x : 8$$
$$\frac{500}{400} = \frac{x}{8}$$
$$x = \frac{500 \times 8}{400} \text{ weeks}$$
$$x = 5 \times 2 \text{ weeks}$$
$$x = 10 \text{ weeks}$$

Hence, the food will last 10 weeks.

6. Since, more speed, less time, so it is a problem of inverse variation. We have the following table :

Speed (in km/h)	72	x
Time (in hours)	10	9

Now,
$$72 : x = 9 : 10$$
$$\frac{72}{x} = \frac{9}{10}$$
$$x = \frac{72 \times 10}{9} \text{ km/h}$$
$$x = 8 \times 10 \text{ km/h}$$
$$x = 80 \text{ km/h}$$

7. More men, less days, so it is a problem of inverse variation. We have the following table :

No. of men	25	x
No. of days	12	6

Now,
$$25 : x = 6 : 12$$
$$\frac{25}{x} = \frac{6}{12}$$
$$x = \frac{12 \times 25}{6}$$
$$x = 2 \times 25 = 50 \text{ men}$$

Hence, the required number of men is 50.

8. Let the required time be x hrs.

Hours	4	x
Days	15	10

Now,
$$4 : x = 10 : 15$$
$$\frac{4}{x} = \frac{10}{15}$$
$$x = \frac{4 \times 15}{10} \text{ hours}$$
$$x = 2 \times 3 \text{ hours}$$
$$x = 6 \text{ hours}$$

Hence, 6 hours a day she should work so as to finish the work in 10 days.

9. Since, more men, less days, so it is a problem of variation. We have the following table :

No. of men	12	x
No. of days	72	54

Now,
$$12 : x = 54 : 72$$
$$\frac{12}{x} = \frac{54}{72}$$
$$x = \frac{72 \times 12}{54}$$
$$x = \frac{8 \times 12}{6}$$
$$x = 8 \times 2 = 16 \text{ men}$$

Hence, 16 men will be required to do the same work in 54 days.

10. Let the required number of persons be x.

Since, more person, less days, so it is a problem of inverse variation. We have the following table :

No. of person	1800	x
No. of days	40	24

Now, $1800 : x = 24 : 40$

$$\frac{1800}{x} = \frac{24}{40}$$
$$x = \frac{1800 \times 40}{24}$$
$$x = \frac{18000}{6}$$
$$= 3000$$
$$x = 3000$$

Hence, 3000 persons are needed to complete the construction of the building in 24 days.

11. Let he takes x days to finish the book.

No. of pages	8	12
No. of days	15	x

Now,
$$8 : 12 = x : 15$$
$$\frac{8}{12} = \frac{x}{15}$$
$$x = \frac{8 \times 15}{12}$$
$$x = \frac{2 \times 15}{3}$$
$$x = 2 \times 5$$
$$x = 10 \text{ days}$$

Hence, he will take 10 days to finish the book.

12. The remaining food would last $(60-10)$ or 50 days for 210 men. But 60 men have left. Number of remaining men $= 210 - 60 = 150$.

For 210 men, food last 50 days. Suppose for 150 men, food last x days.

Less men, more days, so it is a problem of inverse variation. We have the following table :

No. of men	210	150
No. of days	50	x

Now,
$$210 : 150 = x : 50$$
$$\frac{210}{150} = \frac{x}{50}$$
$$x = \frac{210 \times 50}{150} \text{ days}$$
$$x = 70 \text{ days}$$

Hence, the food will last 70 days.

Exercise 5.3

1. We have,

Sharukh can complete the work in 6 days.

$\therefore$ Sharukh's 1 day's work $= \frac{1}{6}$ of the work.

Salman can finish the work in 12 days.

$\therefore$ Salman's 1 day's work $= \frac{1}{12}$ of the work.

$\therefore$ Sharukh's and Salman's 1 day's work

$$= \frac{1}{6} + \frac{1}{12} = \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4} \text{ of the work.}$$

Sharukh and Salman working together in 1 day can finish $\frac{1}{4}$ of the work.

$\therefore$ Sharukh's and Salman's working together can finish the work in $\frac{4}{1} = 4$ days.

2. X can complete the work in 10 days.

$\therefore$ X's 1 day's work $= \frac{1}{10}$ of the work

X and Y can finish the work in 6 days.

$\therefore$ $(X + Y)$'s 1 day's work $= \frac{1}{6}$ of the work

(a) $\therefore$ Y's 1 day's work $= \left(\frac{1}{6} - \frac{1}{10}\right)$ of the work.

$$= \left(\frac{5-3}{30}\right) \text{ of the work.}$$

$$= \left(\frac{2}{30}\right) \text{ of the work.}$$

$$= \frac{1}{15} \text{ of the work.}$$

$\therefore$ Y's alone can finish the work $= 15$ day.

(b) $\because$ Y's 1 day's work $= \frac{1}{15}$ of the work.

$\therefore$ Y's 3 days work $= \frac{3}{15}$ of the work.

$$= \frac{1}{5} \text{ of the work.}$$

$\therefore$ the work left if Y alone works on it for 3 days.

$$= 1 - \frac{1}{5}$$

$$= \frac{5-1}{5} = \frac{4}{5}$$

3. Sandeep can reap a field $= 20$ days.

$\therefore$ Sandeep's 1 day's work $= \frac{1}{20}$

Kuldeep can reap a field $= 30$ days

$\therefore$ Kuldeep's 1 day's work $= \frac{1}{30}$

$\therefore$ (Sandeep + Kuldeep)'s 1 day's work

$$= \left(\frac{1}{20} + \frac{1}{30}\right) = \left(\frac{3+2}{60}\right) = \frac{5}{60} = \frac{1}{12}$$

$\therefore$ Sandeep and Kuldeep working together can complete the work in 12 days.

4. X and Y can do a piece of work in 8 days.

$\therefore$ $(X+Y)$'s 1 day's work $= \frac{1}{8}$

X can do a piece of work in 40 days

$\therefore$ X's 1 day's work $= \frac{1}{40}$

$\therefore$ Y's 1 day's work $= \frac{1}{8} - \frac{1}{40}$

$$= \frac{5-1}{40} = \frac{4}{40} = \frac{1}{10}$$

$\therefore$ Y working can complete the work in 10 days.

5. Time taken by A, B and C to do a piece of work in 10 days.

Time taken by A to do a piece of work in 40 days.

Time taken by B to do a piece of work in 30 days.

$\therefore$ $(A+B+C)$'s 1 day's work $= \frac{1}{10}$

A's 1 day's work $= \frac{1}{40}$

B's 1 day's work $= \frac{1}{30}$

Now, C's 1 day's work $= (A+B+C)$'s 1 days work

− (A's 1 day's work + B's 1 day's work)

B's 1 day's work $= \frac{1}{10} - \left(\frac{1}{40} + \frac{1}{30}\right)$

$$= \frac{12-(3+4)}{120}$$

$$= \frac{12-7}{120} = \frac{5}{120} = \frac{1}{24}$$

Hence, in 24 days, calone will do the same work.

6. A alone can complete a work in 30 days

$\therefore$ A' 1 day's work $= \frac{1}{30}$

$(A+B)$ can complete a work in 20 days

$\therefore$ $(A+B)$'s 1 day's work $= \frac{1}{20}$

$\therefore$ $(A+B)$'s 10 day's work $= 10 \times \frac{1}{20} = \frac{1}{2}$

Remaining work $= 1 - \frac{1}{2} = \frac{1}{2}$

B's 1 day's work $= \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}$

$\therefore$ $$B \text{ finish the work } = \frac{\frac{1}{2}}{\frac{1}{60}} = \frac{60}{2} = 30 \text{ days}$$

Hence, in 30 days, *B* will finish the work.

7. Since, Mandeep can complete a piece of work in 12 days.

$\therefore$ $$\text{Mandeep's 1 day's work } = \frac{1}{12}$$

Since, Sukhvinder can complete a piece of work in 9 days.

$\therefore$ $$\text{Sukhvinder's 1 day's work } = \frac{1}{9}$$

First 4 days,

$$\text{Mandeep's 4 day's work } = 4 \times \frac{1}{12} = \frac{1}{3}$$

Similarly,

$$\text{Sukhvinder's 3 day's work } = 3 \times \frac{1}{9} = \frac{1}{3}$$

(Mandeep + Sukhvinder)'s together work

$$= \left(\frac{1}{3} + \frac{1}{3}\right) = \frac{2}{3}$$

So, the remaining work after 7 days $= 1 - \frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}$

Hence, the remaining work after 7 days $= \frac{1}{3}$.

8. A can complete a piece of work in 6 days.

$\therefore$ $$A\text{'s 1 day's work } = \frac{1}{6}$$

Similarly,

B can complete a piece of work in 4 days

$\therefore$ $$B\text{'s 1 day's work } = \frac{1}{4}$$

$$(A + B)\text{'s 1 days work } = \left(\frac{1}{6} + \frac{1}{4}\right)$$

$$A\text{'s 2 days work } = 2 \times \frac{1}{6} = \frac{1}{3}$$

Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$ (The work which $(A + B)$ will do together)

$\therefore$ $$\left(\frac{1}{6} + \frac{1}{4}\right) : \frac{2}{3} = 1 : x$$

$$\frac{5}{12} : \frac{2}{3} = 1 : x$$

$$\frac{\frac{5}{12}}{\frac{2}{3}} = \frac{1}{x}$$

$$x = \frac{2\times 12}{5\times 3} = \frac{2\times 4}{5} = \frac{8}{5} = 1\frac{3}{5} \text{ days}$$

Hence, $1\frac{3}{5}$ days will be needed more to complete that work.

9. $(A+B)$ can complete a piece of work in 8 days.

$\therefore$ $(A+B)$'s 1 day's work $= \frac{1}{8}$

Similarly,

$(B+C)$'s 1 day's work $= \frac{1}{12}$

Also, $(A+B+C)$'s 1 day's work $= \frac{1}{6}$

$\therefore$ $(A+C)$'s 1 day's work $= 2$ $(A+B+C)$'s 1 day's work $-(A+B)$'s 1 day's work $-(B+C)$'s 1 day's work

$$= \frac{2}{6} - \left[\frac{1}{8} + \frac{1}{12}\right]$$

$$= \frac{2}{6} - \left[\frac{3+2}{24}\right]$$

$$= \frac{2}{6} - \frac{5}{24} = \frac{8-5}{24} = \frac{3}{24} = \frac{1}{8}$$

$\therefore$ $(A+C)$ working together can complete the work in 8 days.

10. One tap fills a tank in 20 minutes.

$\therefore$ tap one's 1 minute of filling the tap $= \frac{1}{20}$

Other tap fills a tank in 12 minutes

$\therefore$ tap other's 1 minute of filling the tank $= \frac{1}{12}$

In 1 minute work done by both tap $= \frac{1}{20} + \frac{1}{12}$

$$= \frac{3+5}{60} = \frac{8}{60}$$

$$= \frac{2}{15} \text{ part of tank}$$

Thus, the time taken to fill the tank if both taps are opened

$$= \frac{15}{2} = 7\frac{1}{2} \text{ min.}$$

11. Salma's 1 day's work $= \frac{1}{6}$

Ali's 1 day's work $= \frac{1}{10}$

Pervej's 1 day's work $= \frac{1}{15}$

$\therefore$ (Salma + Ali)'s 1 day's work $= \frac{1}{6} + \frac{1}{10} = \frac{10+6}{60}$

$= \frac{16}{60} = \frac{4}{15}$

$\therefore$ (Salma + Ali)'s 3 days work $= \frac{3 \times 4}{15} = \frac{4}{5}$

So, remaining work $= 1 - \frac{4}{5} = \frac{1}{5}$

(Ali + Pervej)'s 1 day's work $= \frac{1}{10} + \frac{1}{15}$

$= \frac{3+2}{30} = \frac{1}{6}$

$\therefore$ (Ali + Pervej) can finish the remaining work $= \frac{\frac{1}{5}}{\frac{1}{6}} = \frac{6}{5} = 1\frac{1}{5}$ days.

12. A tap fills a water tank in 8 hours

$\therefore$ tan's 1 minute of filling the water tank $= \frac{1}{8}$

A pipe empties a tank in 12 hours

$\therefore$ It will make empty in minute $= \frac{1}{12}$

In 1 minute work done by both the tap and pipe$= \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$

Thus, the time taken to fill the cistern if both the tap and the pipe are opened together = 24 hours.

13. 2 km of bridge can be completed by workers = 42

$\therefore$ 1 km of bridge can be completed by wokers $= \frac{42}{2}$

$\therefore$ 9 km of bridge can be completed by worker $= \frac{42}{2} \times 9 = 189$ worker

So, the required number of worker $= 189 - 42 = 147$

Hence, 147 more workers should be employed.

14. Since, Vani can complete $\frac{1}{3}$ or the work in 10 hours.

$\therefore$ Vani's one hour's work $= \frac{1}{10}$ of $\frac{1}{3} = \frac{1}{10} \times \frac{1}{3} = \frac{1}{30}$

Since, Devesh can complete $\frac{1}{5}$ of the same work in 12 hours.

$\therefore$ Devesh's one hour's work $= \frac{1}{12}$ of $\frac{1}{5} = \frac{1}{12} \times \frac{1}{5} = \frac{1}{60}$

$\therefore$ (Vani + Devesh)'s 1 hour's work $= \frac{1}{60} + \frac{1}{30} = \frac{1+2}{60} = \frac{3}{60} = \frac{1}{20}$

Hence, Vani and Devesh working together can complete the work in 20 hours.

15. Since Ram, Shyam and Mukesh can reap a field in $\frac{63}{4}$ days.

$\therefore$ (Ram + Shyam + Mukesh)'s 1 day's work $= \frac{4}{63}$

Since, Shyam, Mukesh and Chandu can reap a field in 14 days.

$\therefore$ (Shyam + Mukesh + Chandu)'s 1 day's work $= \frac{1}{14}$

Since, Mukesh, Chandu and Ram can reap a field in 18 days.

$\therefore$ (Makesh + Chandu + Ram)'s 1 day's work $= \frac{1}{18}$

And

Since, Chandu, Ram and Shyam can reap a field in 21 days.

$\therefore$ (Chandu + Ram + Shyam)'s 1 day's work $= \frac{1}{21}$

$\therefore$ 3 (Ram + Shyam + Mukesh + Chandu)'s 1 day's work

$$= \left(\frac{4}{63} + \frac{1}{14} + \frac{1}{18} + \frac{1}{21}\right)$$

$$= \left(\frac{4 \times 2 + 9 + 7 + 6}{126}\right)$$

$$= \frac{30}{126}$$

$\therefore$ (Ram + Shyam + Mukesh + Chandu)'s 1 day's work $= \frac{1}{3} \times \frac{30}{126} = \frac{10}{126} = \frac{5}{63}$

Hence, (Ram + Shyam + Mukesh + Chandu) working together can complete the work in $\frac{63}{5}$ or $12\frac{3}{5}$ days.

Exercise 5.4

1. (a) $\because$ 1 km/h $= \frac{5}{18}$ m/sec

$\therefore$ 48 km/h $= 48 \times \frac{5}{18}$ m/sec $= 13.33$ m/sec

(b) $\because$ 1 km/h $= \frac{5}{18}$ m/sec

$\therefore$ 126 km/h $= 126 \times \frac{5}{18}$ m/sec

$= 7 \times 5$ m/sec $= 35$ m/sec

(c) $\because$ 1 m/sec $= \frac{18}{5}$ km/h

$\therefore$ 36 m/sec $= 36 \times \frac{18}{5}$ km/h $= 129.6$ km/h

(d) $\because$ 1 m/sec $= \frac{18}{5}$ km/h

$\therefore$ 63 m/sec $= 63 \times \frac{18}{5}$ km/h $= 226.8$ km/h

2. Speed of motor boat $= \frac{36}{5}$ km/h

$\therefore$ Speed of motor boat in m/sec $= \frac{36}{5} \times \frac{5}{18}$ m/sec $= 2$ m/sec

3. Distance $= 90$ km, Time $= 1$ hour 30 min

$= 90 \times 1000$ m $\quad = (60 + 30)$ min

$= 90000$ m $\quad = 90$ min $= 90 \times 60$ sec

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{90000}{90 \times 60} \text{ m/sec}$$

$$= \frac{90000}{5400} \text{ m/sec.} = 16.66 \text{ m/s}$$

4. Speed $= 72$ km/h, Time $= 18$ sec

$$= 72 \times \frac{5}{18} \text{ m/sec}$$

$$= 4 \times 5 \text{ m/sec}$$

$$= 20 \text{ m/sec}$$

So, distance covered by train $=$ Speed $\times$ Time $= 20 \times 18 = 360$ m

5. Since, side of a square $= 35$ m

$\therefore$ the perimeter of a square $= 4 \times$ side of a square

$$= 4 \times 35 \text{ m}$$

$$= 140 \text{ m}$$

Since, the perimeter of a square is equal to distance covered by a boy.

$$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{140}{7.2 \times \frac{5}{18}}$$

$$= \frac{140 \times 18}{7.2 \times 5}$$

$$= \frac{2520}{36} = 70 \text{ sec or 1 min 10 sec}$$

6. Since, 72 km/h $= 72 \times \frac{5}{18}$ m/sec

$$= 4 \times 5 \text{ m/sec} = 20 \text{ m/sec}$$

$\therefore$ 20 m/sec > 18 m/sec

For 20 m/sec,

$$\text{Distance}_1 = \text{Sped} \times \text{Time}$$

$$= 20 \times 2 = 40 \text{ m}$$

Also,

For 18 m/sec,

$$\text{Distance}_2 = 18 \times 2 = 36 \text{ m}$$

$$\text{Difference} = d_1 - d_2 = (40 - 36) \text{ m} = 4 \text{ m}$$

7. The length of the train $= 210$ m

Time $= 7$ sec

$$\text{Speed} = \frac{\text{length of the train}}{\text{Time}}$$

$$= \frac{210}{7} = 30\,\text{m/sec.}$$

Hence, the speed of the train is 30 m/sec.

8. $\text{length of train} = 450\,\text{m}$

$\text{Time} = 15\,\text{sec.}$

$\therefore$ $\text{Speed of train} = \frac{450}{15} = 30\,\text{m/sec}$

Hence, the speed of the train is 30 m/sec.

9. $\text{length of train} = 225\,\text{m}$

$\text{Crossing time of a man} = 10\,\text{sec.}$

$\therefore$ $\text{Speed of train} = \frac{225}{10} = 22.5\ \text{m/sec.}$

$\text{If the length of a platform} = 405\,\text{m}$

$\therefore$ $\text{passing time of a train} = \frac{405}{22.5} = 18\,\text{sec.}$

10. Let the usual speed and usual time be x m/sec and y sec respectively.

$$\text{Distance} = (xy)\,\text{m} \qquad \ldots\text{(i)}$$

When Anil runs at $\frac{5}{4}$th of his usual speed. Then, same distance covered by Anil.

$$\text{Distance} = \frac{5}{4}x \times (y-5)$$

$$xy = \frac{5}{4}x \times (y-5)\ \text{[from equation (i)]}$$

$$y = \frac{5}{4}(y-5)$$

$$4y = 5y - 25$$

$$5y - 4y = 25$$

$$y = 25\,\text{min}$$

Hence, his usual time is 25 min.

11. Since, the trains are going in the opposite direction, therefore, the relative speed will be given by sum of their speeds.

$\therefore$ $\text{relative speed} = (55+53)\,\text{km/h}$

$$= 108\,\text{km/h}$$

$$= 108 \times \frac{5}{18}\ \text{m/sec}$$

$$= 30\,\text{m/sec.}$$

Distance covered by the trains to pass each other

$$= (350+420)\,\text{m}$$

$$= 770\,\text{m}$$

$\therefore$ Time taken by two train to pass each other

$$= \frac{770}{30} = 25.66\ \text{sec.}$$

12.

$$\begin{aligned}
\text{Speed of train} &= 45\,\text{km/hrs} = 45 \times \frac{5}{18}\,\text{m/s} = \frac{225}{18}\,\text{m/s}\\
\text{Time} &= 18\,\text{seconds}\\
\therefore \quad \text{the length of a train} &= \text{Speed} \times \text{Time}\\
&= \left(45 \times 18 \times \frac{5}{18}\right)\text{m} = 225\,\text{m}\\
\therefore \quad \text{total distance} &= (225+125)\,\text{m} = 350\,\text{m}\\
\therefore \quad \text{time taken in crossing the platform} &= \frac{\text{Total distance}}{\text{Speed}}\\
&= \left(\frac{350}{225} \times 18\right)\text{sec.}\\
&= (1.55 \times 18)\ \text{sec.}\\
&= 27.99\ \text{sec.} = 28\,\text{sec.}
\end{aligned}$$

13. Since, the trains are going in the same direction, therefore, the relative speed will be given by the difference of their speeds.

$$\begin{aligned}
\therefore \quad \text{relative speed} &= (50-45)\ \text{km/h}\\
&= 4\ \text{km/h} = 4 \times \frac{5}{18}\ \text{m/sec}\\
&= \frac{20}{18}\ \text{m/sec}
\end{aligned}$$

Distance covered by the trains to pass each other $= (310+360)\,\text{m} = 670\,\text{m}$

$$\begin{aligned}
\therefore \quad \text{time taken to cross each other} &= \frac{\text{Distance covered}}{\text{Relative speed}}\\
&= \left(\frac{670}{20} \times 18\right)\text{sec.}\\
&= (67 \times 9)\,\text{sec}\\
&= 603\,\text{sec}
\end{aligned}$$

MCQs

1. (c) **2.** (b) **3.** (c) **4.** (c) **5.** (b) **6.** (a)

6

Percentage and its Applications

Exercise 6.1

1. (a) $3\frac{2}{5}\% = \frac{3 \times 5 + 2}{5}\% = \frac{15+2}{5}\% = \frac{17}{5} \times \frac{1}{100} = \frac{17}{500}$

(b) $69\% = 69 \times \frac{1}{100} = \frac{69}{100}$

(c) $5\frac{1}{4} = \frac{21}{4} = \left(\frac{21}{4} \times 100\right)\% = (21 \times 25)\% = 525\%$

(d) $72 : 9 = \frac{72}{9} = \left(\frac{72}{9} \times 100\right)\% = (8 \times 100)\% = 800\%$

2. (a)
$$6\% \text{ of } x = 18$$
$$\frac{6}{100} \times x = 18$$
$$x = \frac{18 \times 100}{6} = 3 \times 100 = 300$$

(b)
$$10\% \text{ of } x = 31$$
$$\frac{10}{100} \times x = 31$$
$$x = \frac{31 \times 100}{10}$$
$$= 31 \times 10 = 310$$

(c)
$$12\% \text{ of } x = 36$$
$$\frac{12}{100} \times x = 36$$
$$x = \frac{36 \times 100}{12} = 3 \times 100 = 300$$

(d)
$$\frac{1}{3}\% \text{ of } x = 21$$
$$\frac{1}{3 \times 100} \times x = 21$$
$$\frac{x}{300} = 21$$
$$x = 21 \times 300 = 6300$$

(e)
$$5\% \text{ of } x = 6$$
$$\frac{5}{100} \times x = 6$$
$$x = \frac{6 \times 100}{5} = 6 \times 20 = 120$$

(f)
$$7.6\% \text{ of } x = 152$$
$$\frac{7.6}{100} \times x = 152$$
$$x = \frac{152 \times 100}{7.6}$$
$$= \frac{15200}{7.6} = 2000$$

3. (a) $\left(\frac{20}{80} \times 100\right)\% = \left(\frac{1}{4} \times 100\right)\% = 25\%$

(b) $\left(\frac{30}{600} \times 100\right)\% = \left(\frac{30}{6}\right)\% = 5\%$

(c) $\left(\frac{300}{5 \times 1000} \times 100\right)\% = \left(\frac{30000}{5000}\right)\% = \left(\frac{30}{5}\right)\% = 6\%$

(d) $\left(\frac{8.75}{7} \times 100\right)\% = 125\%$

(e) $\left(\dfrac{\frac{1}{3}}{\frac{2}{9}}\times 100\right)\% = \left(\dfrac{9}{3\times 2}\times 100\right)\% = 150\%$

(f) $\left(\dfrac{\frac{1}{20}}{\frac{1}{16}}\times 100\right)\% = \left(\dfrac{16}{20}\times 100\right)\% = (16\times 5)\% = 80\%$

4. (a) Let the required number be x.

Then,

$$55\% \text{ of } x = 11$$

$$\frac{55}{100}\times x = 11$$

$$x = \frac{11\times 100}{55} = \frac{100}{5} = 20$$

Hence, the number is 20.

(b) Let the required number be y.

Then,

$$12\frac{1}{2}\% \text{ of } y = 15$$

$$\frac{25}{2}\%\times y = 15$$

$$\frac{25}{2}\times\frac{1}{100}\times y = 15$$

$$\frac{1}{8}\times y = 15$$

$$y = 15\times 8 = 120$$

Hence, the required number is 120.

(c) Let the number be x.

Then,

$$13.25\% \text{ of } x = 318$$

$$13.25\times\frac{1}{100}\times x = 318$$

$$x = \frac{318\times 100}{13.25} = \frac{31800}{13.25} = 2400$$

Hence, the number is 2400.

5. Let the original price of the ticket be ₹ x.

Then,

$$x + 20\% \text{ of } x = ₹\ 600$$

$$x + \frac{20}{100}\times x = ₹\ 600$$

$$x + \frac{1}{5}x = ₹\ 600$$

$$\frac{5x + x}{5} = ₹\ 600$$

$$\frac{6x}{5} = ₹\ 600$$

$$x = ₹\ \frac{600\times 5}{6}$$

x = ₹ 100×5 = ₹ 500

Hence, the original price of the ticket is ₹ 500.

6. The original cost of a motorcycle = ₹ 40000

Decrease percentage $= 22\%$

$\therefore$ the new value of motorcycle = ₹ $\left(40000 - \frac{22}{100} \times 40000\right)$

= ₹ $(40000 - 22 \times 400)$

= ₹ $(40000 - 8800)$

= ₹ 31200

Hence, the new value of a motorcycle is ₹ 31200.

7. The total number of trees $= 64$

Then, the number of apple trees $= 64 \times \frac{25}{100}$

$= \frac{64}{4} = 16$

Also, the number of guava trees $= 64 \times 62.5\%$

$= 64 \times \frac{62.5}{100}$

$= \frac{64 \times 62.5}{100}$

$= \frac{4000}{100} = 40$

The percentage of banana trees $= (100 - 25 - 62.5)\%$

$= 12.5\%$

$\therefore$ the number of banana trees $= 64 \times 12.5\%$

$= 64 \times \frac{12.5}{100} = \frac{800}{100} = 8$

Hence, the number of apple trees, guava trees and banana trees are 16, 40 and 8 respectively in the orchard.

8. The total numbers of votes $= 600000$

$\therefore$ the numbers of invalid votes $= 600000 \times 8\%$

$= 600000 \times \frac{8}{100} = 48000$

$\therefore$ the number of valid votes $= 600000 - 48000 = 552000$

So, the number of valid polled in favour of the candidate

$= 552000 \times \frac{65}{100}$

$= 5520 \times 65 = 358800$

9. Let the number be 100.

20% increases means that the number becomes

$= (100 + 20) = 120$

Decreased number $= \frac{20}{100} \times 120 = 24$

i.e., $120 - 24 = 96$

$\therefore$ decreased percentage $= \dfrac{\text{Change in number}}{\text{Original number}}$

$$= \left(\frac{100-96}{100} \times 100 \right)\% = 4\%$$

10. The percentage of charcoal $= (100-75-10)\%$

$$= (100-85)\% = 15\%$$

$\therefore$ the amount of charcoal $= (9 \times 15\%)\,\text{kg}$

$$= \left(9 \times \frac{15}{100} \right) \text{kg}$$

$$= \frac{9 \times 3}{20} \text{kg}$$

$$= \frac{27}{20} \text{kg} = 1.35 \text{ kg}$$

or $1.35 \times 1000 \text{ g} = 1350 \,\text{gm}$

Hence, the amount of charcoal is 1350 gm in 9 kg of gun-powder.

11. Let the original number of sheep be x.

Then, 10% sheep are died

So, Remaining number of sheep $= x - x \times 10\%$

$$= x - \frac{x \times 10}{100} = \frac{9x}{10}$$

But, 40% of the remaining died due to a disease.

So, remaining number of sheep $= \dfrac{9x}{10} - \dfrac{9x}{10} \times \dfrac{40}{100}$

$$= \frac{9x}{10} - \frac{9x}{25} = \frac{45x - 18x}{50} = \frac{27x}{50}$$

Therefore, $\dfrac{27x}{50}$ is the number of alive sheep.

So, $\dfrac{27x}{50} = 594$

$$x = \frac{594 \times 50}{27} = 22 \times 50 = 1100$$

Hence, the original number of sheep is 1100 in the farm.

12. Let the original price of an article $=$ ₹ 100

Decrease in price of an article $= 10\%$ of ₹ 100 $= \dfrac{10}{100} \times$ ₹ 100 $=$ ₹ 10

So, the new price of an article $=$ ₹ (100–10) $=$ ₹ 90

Now, the new price by increased to retain price

$$= \left(\frac{10}{90} \times 100 \right)\% = \frac{1000}{90}\% = 11.11\%$$

13. Let the original length be x cm.

Then, $x + x \times 24\% = 31 \text{cm}$

$$x + \frac{24}{100} x = 31 \text{cm}$$

$$
\begin{aligned}
x+\frac{6x}{25} &= 31\,\text{cm}\\
\frac{25x+6x}{25} &= 31\,\text{cm}\\
\frac{31x}{25} &= 31\,\text{cm}\\
31x &= 31\times 25\,\text{cm}\\
x &= \frac{31\times 25}{31}\,\text{cm}\\
x &= 25\,\text{cm}
\end{aligned}
$$

Hence, the unstretched original length is 25 cm.

14. The obtained marks in half yearly

examination $= \text{Percentage} \times \text{maximum marks}$

$= 90\% \times 1000 = \frac{90}{100}\times 1000 = 900$

Similarly,

The obtained marks in annual examination $= \text{percentage}\times \text{maximum marks}$

$= 84\% \times 1200$

$= \frac{84}{100}\times 1200 = 84\times 12 = 1008$

The sum of total obtained marks $= 900 + 1008 = 1908$

$\therefore$ the percentage of marks by Yash $= \left(\frac{1908}{1000+1200}\right)\times 100\%$

$= \left(\frac{1908}{2200}\times 100\right)\% = \frac{1908}{22}\% = 86.72\%$

15. Let the total number of passengers be x.

The number of passengers at station B

$= x - 40\%\text{ of } x = x - \frac{40}{100}\times x = \frac{3x}{5}$

Also, the number of passengers at station $= 12$

$$
\begin{aligned}
\therefore\quad \frac{3x}{5} - 75\%\text{ of }\frac{3x}{5} &= 12\\
\frac{3x}{5} - \frac{75}{100}\times\frac{3x}{5} &= 12\\
\frac{3x}{5} - \frac{3}{4}\times\frac{3x}{5} &= 12\\
\frac{3x}{5} - \frac{9x}{20} &= 12\\
\frac{3x\times 4 - 9x}{20} &= 12\\
\frac{12x-9x}{20} &= 12\\
3x &= 12\times 20\\
x &= 4\times 20\\
x &= 80
\end{aligned}
$$

Hence, the total number of passengers is 80.

16. Let the original price of land be ₹ x.

Then,
$$\begin{aligned} x+20\% \text{ of } x &= ₹\ 1500000. \\ x+\frac{20}{100}\times x &= ₹\ 1500000 \\ x+\frac{x}{5} &= ₹\ 1500000 \\ \frac{5x+x}{5} &= ₹\ 1500000 \\ \frac{6x}{5} &= ₹\ 1500000 \\ x &= ₹\ \frac{1500000\times 5}{6} \\ x &= ₹\ 250000\times 5 \\ x &= ₹\ 1250000 \end{aligned}$$

Hence, the original price of land is ₹ 125000.

17. Let the maximum marks be x.

For A,
$$\begin{aligned} \text{passing marks} &= 30\% \text{ of } x+40 \\ &= \frac{30}{100}\times x+40 \end{aligned}$$

For B,
$$\begin{aligned} \text{passing marks} &= 40\% \text{ of } x-20=\frac{40}{100}\times x-20 \\ \frac{30}{100}x+40 &= \frac{40}{100}x-20 \\ \frac{40}{100}x-\frac{30}{100}x &= 40+20 \\ \frac{40x-30x}{100} &= 60 \\ \frac{10}{100}x &= 60 \\ x &= 60\times 10 \\ x &= 600 \end{aligned}$$

Hence, the maximum marks is 600.

$$\begin{aligned} \text{The minimum pass marks} &= \frac{30}{100}\times 600+40 \\ &= 30\times 6+40=180+40=220 \text{ marks} \end{aligned}$$

18. Let the price of a table be ₹ x without including to x.

Then,
$$\begin{aligned} x+8\% \text{ of } x &= ₹\ 4914 \\ x+\frac{8}{100}\times x &= ₹\ 4914 \\ x+\frac{2x}{25} &= ₹\ 4914 \\ \frac{25x+2x}{25} &= ₹\ 4914 \end{aligned}$$

$$\begin{aligned} \frac{27}{25}x &= ₹\ 4914 \\ x &= ₹\ \frac{4914 \times 25}{27} \\ x &= ₹\ 182 \times 25 = ₹\ 4550 \end{aligned}$$

Hence, the price of the table is ₹ 4550 without any tax.

19. Let her monthly income be ₹ x.

$$\begin{aligned} \text{The percentage of spent} &= (100-10)\% \\ &= 90\% \\ \text{Now,} \quad \text{10\% of income of 6 month} &= ₹\ 4800 \\ \therefore \quad \text{10\% of income of per month} &= ₹\ 4800 \div 6 = ₹\ 800 \\ \therefore \quad \text{total income} &= ₹\left(\frac{800}{10} \times 100\right) \\ &= ₹\ 800 \times 10 = ₹\ 8000 \end{aligned}$$

Hence, the total income of Mrs Khurana is ₹ 8000.

20. Let his salary be ₹ x.

$$\begin{aligned} \text{Then,} \quad x - 20\% \text{ of } x &= ₹\ 6560 \\ x - \frac{20}{100} \times x &= ₹\ 6560 \\ x - \frac{1}{5}x &= ₹\ 6560 \\ \frac{5x - x}{5} &= ₹\ 6560 \\ \frac{4}{5}x &= ₹\ 6560 \\ x &= ₹\ \frac{6560 \times 5}{4} \\ x &= ₹\ \frac{32800}{4} \\ x &= ₹\ 8200 \end{aligned}$$

Hence, his salary is ₹ 8200.

Exercise 6.2

1.

$$\begin{aligned} \text{CP of a TV set} &= ₹\ 12500 \\ \text{SP of a TV set} &= ₹\ 13000 \end{aligned}$$

Since, SP > CP

$$\begin{aligned} \therefore \quad \text{gain} &= \text{SP} - \text{CP} = ₹\ (1300 - 12500) = ₹\ 500 \\ \text{gain }\% &= \frac{500}{12500} \times 100 \left[\because \text{gain}\% = \frac{\text{gain}}{\text{CP}} \times 100\right] \\ &= 4\% \end{aligned}$$

Hence, the gain and gain% of a TV set is ₹ 500 and 4% respectively.

2.

$$\begin{aligned} \text{The cost of 25 pens} &= ₹\ 25 \times 8 = ₹\ 200 \\ \text{The cost of 35 pens} &= ₹\ 35 \times 10 = ₹\ 350 \\ \therefore \quad \text{the total CP of 60 pens} &= ₹\ (200 + 350) \end{aligned}$$

= ₹ 550

The selling price of 60 pens = ₹ $60 \times 11 =$ ₹ 660

Since, SP > CP

$\therefore$ gain percentage $= \left(\dfrac{SP - CP}{CP} \times 100\right)\%$

$= \left(\dfrac{660 - 550}{550} \times 100\right)\%$

$= \left(\dfrac{110}{550} \times 100\right)\% = \left(\dfrac{1}{5} \times 100\right)\% = 20\%$

3. The total CP of old TV set = ₹ $(8700 + 1100)$

= ₹ 9800

profit % = 8% SP = ?

$\because$ profit % $= \dfrac{SP - CP}{CP} \times 100$

$8 = \dfrac{SP - 9800}{9800} \times 100$

$8 \times 98 = SP - 9800$

SP = ₹ $(9800 + 784)$

SP = ₹ 10584

4. The CP of 1 dozen or 12 of bananas = ₹ 15

The SP of 1 dozen of 12 of bananas = ₹ 20

Since, SP > CP

$\therefore$ gain % $= \left(\dfrac{SP - CP}{CP} \times 100\right)\%$

$= \left(\dfrac{20 - 15}{15} \times 100\right)\% = \left(\dfrac{5}{15} \times 100\right)\%$

$= \dfrac{100}{3}\% = 33\dfrac{1}{3}\%$

5. SP of a coat = ₹ 1020

loss% = 15%

CP of a coat = ?

$\because$ loss % $= \dfrac{CP - SP}{CP} \times 100$

$15 = \dfrac{CP - 1020}{CP} \times 100$

$15\ CP = 100\ CP - 102000$

$100\ CP - 15\ CP = 102000$

$85\ CP = 102000$

$CP = 102000 \div 85$

CP = ₹ 1200

Hence, the CP of the coat is ₹ 1200.

6. Let the cost price of one article be ₹ x.

$\therefore$ CP of 16 articles = ₹ $16\,x$

SP of 12 articles = CP of 16 article = ₹ $16\,x$

$$
\begin{aligned}
\therefore \quad \text{SP of one article} &= ₹\ \frac{16x}{12} \\
&= ₹\ \frac{4}{3}x \\
\therefore \quad \text{Gain} &= \text{SP of 1 article} - \text{CP of 1 article} \\
&= ₹\left(\frac{4}{3}x - x\right) \\
&= ₹\ \frac{1}{3}x \\
\therefore \quad \text{Gain \%} &= \frac{\text{Gain}}{\text{CP}} \times 100\% \\
&= \frac{\frac{1}{3}x}{x} \times 100\% \\
&= \frac{100}{3}\% \\
&= 33\frac{1}{3}\%
\end{aligned}
$$

7.

$$
\begin{aligned}
\text{SP of a sofa set} &= ₹\ 7200 \\
\text{Loss\%} &= 20\% \\
\text{CP of a sofa set} &= ? \\
\because \quad \text{Loss \%} &= \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100 \\
20 &= \frac{\text{CP} - 7200}{\text{CP}} \times 100 \\
\text{CP} &= 5\,\text{CP} - ₹\ 36000 \\
5\ \text{CP} - \text{CP} &= ₹\ 36000 \\
4\ \text{CP} &= ₹\ 36000 \\
\text{CP} &= ₹\ 36000 \div 4 \\
&= ₹\ 9000
\end{aligned}
$$

For making 20% gain,

$$
\begin{aligned}
\text{The new SP of a sofa set} &= (\text{gain\%} \times \text{CP} + \text{CP}) \\
&= 20\% \times 9000 + 9000 \\
&= ₹\left(\frac{20}{100} \times 9000 + 9000\right) \\
&= ₹\ (1800 + 9000) \\
&= ₹\ 10800
\end{aligned}
$$

8. If two items are sold at same SP, one at a loss of $x\%$ and other at a gain of $x\%$, then there is a loss of $\frac{x^2}{100}\%$.

$$
\begin{aligned}
\therefore \quad \text{Loss \%} &= \frac{20^2}{100}\% \\
&= \frac{400}{100}\% = 4\%
\end{aligned}
$$

9. Let the CP_1 of one refrigerator be ₹ x. Then, CP_2 of second refrigerator

$$= ₹\ (30000 - x)$$

And let $SP_1 = ₹\ y$, $SP_2 = ₹\ z$

According to the questions,

$$6 = \frac{x - y}{x} \times 100$$

$$6x = 100x - 100y$$

$$y = \frac{47}{50}x \qquad \text{...(i)}$$

Similarly,

$$12 = \frac{z - (30000 - x)}{30000 - x)} \times 100$$

$$3 = \frac{z - (30000 - x)}{(30000 - x\,x)} \times 25$$

$$90000 - 3x = 25z - 750000 + 25x$$

$$28x + 25z = ₹\ 840000 \text{ ...(ii)}$$

Since, in this deal, no gain or loss.

$$\text{gain} = CP_1 - SP_1 \text{ ...(iii)}$$

$$\text{Loss} = SP_2 - CP_2 \text{ ...(iv)}$$

Equation (iii) & (iv) are equal to each other.

$$CP_1 - SP_1 = SP_2 - CP_2$$

$$CP_1 + CP_2 = SP_1 + SP_2$$

$$₹\ 30000 = y + z$$

$$z = ₹\ (30000 - y)$$

Multiplying both sides by 25

$$25z = ₹\ (30000 - y) \times 25$$

$$25z = ₹\ 750000 - 25y$$

Putting the value of $25z$ in equation (ii), we get

$$28x + ₹\ 750000 - 25y = ₹\ 840000$$

$$28x - 25 \times \frac{47}{50}x = ₹\ 840000 - ₹\ 750000 \text{ [From eq}^{n}\text{ 1]}^{0}$$

$$28x - \frac{47}{2}x = ₹\ 90000$$

10.

$$\text{Loss \%} = \left(\frac{4}{36+4} \times 100\right)\%$$

$$= \left(\frac{4}{40} \times 100\right)\%$$

$$= 10\%$$

11.

$$\text{Total CP of a computer} = ₹\ (16800 + 1200)$$

$$= ₹\ 18{,}000$$

$$\text{SP of a computer} = ₹\ 19{,}200$$

Since, SP > CP

$$\therefore \qquad \text{Gain\%} = \left(\frac{\text{SP} - \text{CP}}{\text{CP}} \times 100\right)\%$$

$$= \left(\frac{19200 - 18000}{18000} \times 100\right)\%$$

$$= \left(\frac{1200}{180}\right)\%$$

$$= \frac{120}{18}\%$$

$$= 6\frac{2}{3}\%$$

12. Let the CP of A be ₹ x.

Then, the CP of B $= x + 20\%$ of x

$$= x + \frac{20}{100}x$$

$$= \frac{6x}{5}$$

$\therefore$ the CP of C $=$ ₹ 441

$$\frac{6x}{5} + \frac{6x}{5} \times \frac{5}{100} = ₹\ 441$$

$$\frac{6x}{5} + \frac{3x}{50} = ₹\ 441$$

$$\frac{60x + 3x}{50} = ₹\ 441$$

$$\frac{63}{50}x = ₹\ 441$$

$$= ₹\ \frac{441 \times 50}{63}$$

$$x = ₹\ 7 \times 50$$

$$x = ₹\ 350$$

Hence, the CP of a calculator is ₹ 350.

13. The CP of 1 dozen or 12 eggs $=$ ₹ 18

$\therefore$ The CP of 1 egg $= ₹\ \frac{18}{12} = ₹\ \frac{3}{2}$

$\therefore$ The CP of 25 dozen or 300 eggs $= ₹\ 18 \times 25 = ₹\ 450$

$\therefore$ The CP of 150 eggs $= ₹\ \frac{3}{2} \times 150 = ₹\ 225.$

$\therefore$ the SP of 150 eggs $= ₹\left(\frac{20 \times 225}{100} + 225\right)$

$$= ₹\ (45 + 225) = ₹\ 270$$

Making gain 30% on his out lay

The total SP of 300 eggs $= ₹\left(\frac{30 \times 450}{100} + 450\right)$

$$= ₹\ (135 + 450) = ₹\ 585$$

$\therefore$ the SP of remaining 150 eggs $= ₹\ (585 - 270)$

$$= ₹\ (315)$$

$\therefore$ the SP of one egg $= ₹\ \frac{315}{150} = ₹\ 2.1.$

Hence, the SP of each egg is ₹ 2.1.

14. (a) The given,

$$\text{MP} = ₹\ 210, \text{ discount } \% = 10\%$$

$$\therefore \quad \text{SP} = \text{MP} - \text{MP of discount}\%$$

$$= ₹\left(210 - 210 \times \frac{10}{100}\right)$$

$$= ₹\ (210 - 21) = ₹\ 189$$

(b) The given,

$$\text{MP} = ₹\ 1500, \text{ discount}\% = 12\frac{1}{2}\%$$

$$\therefore \quad \text{SP} = ₹\left(1500 - 1500 \times \frac{25}{2} \times \frac{1}{100}\right)$$

$$= ₹\ (1500 - 187.5) = ₹\ (1312.5)$$

(c) The given,

$$\text{MP} = ₹\ 1200, \text{ discount } \% = 25\%$$

$$\therefore \quad \text{SP} = ₹\left(1200 - 1200 \times \frac{25}{100}\right)$$

$$= ₹\ (1200 - 300) = ₹\ 900$$

15. (a) The given,

$$\text{SP} = ₹\ 1080, \text{ discount } \% = 25\%$$

$$\therefore \quad \text{MP} = \text{SP} + \text{discount}$$

$$\text{Discount } \% = \frac{\text{discount}}{\text{MP}} \times 100$$

$$25 = \frac{\text{discount}}{\text{SP} + \text{discount}} \times 100$$

$$25 = \frac{\text{discount}}{1080 + \text{discount}} \times 100$$

$$1080 = 3 \text{ discount}$$

$$\text{discount} = ₹\ 360$$

$$\therefore \quad \text{MP} = \text{SP} + \text{discount}$$

$$= ₹\ (1080 + 360) = ₹\ 1440$$

(b)

$$\text{Discount } \% = \frac{\text{discount}}{\text{SP} + \text{discount}} \times 100$$

$$15 = \frac{\text{discount}}{552.50 + \text{discount}} \times 100$$

$$3 = \frac{20 \text{ discount}}{552.50 + \text{discount}}$$

$$₹\ 1657.50 + 3 \text{ discount} = 20 \text{ discount}$$

$$17 \text{ discount} = ₹\ 1657.50$$

$$\text{discount} = ₹\ 97.50$$

$$\therefore \quad \text{MP} = ₹\ (552.50 + 97.50)$$

$$\text{MP} = ₹\ 650$$

16. (a) The given,

$$\text{MP} = ₹\ 1600,\ \text{SP} = ₹\ 1280$$

$$\begin{aligned}\text{discount} &= \text{MP} - \text{SP}\\ &= ₹\ (1600-1280)\\ &= ₹\ 320\end{aligned}$$

$$\text{discount\%} = \left(\frac{\text{discount}}{\text{MP}} \times 100\right)\% = \left(\frac{320}{1600} \times 100\right)\% = \left(\frac{320}{16}\right)\% = 20\%$$

(b) The given,

$$\text{MP} = ₹\ 625.50,\ \text{SP} = ₹\ 562.95$$

$$\text{discount} = \text{MP} - \text{SP} = ₹\ (625.50 - 562.95) = ₹\ 62.55$$

\

$$\text{Discount\%} = \left(\frac{\text{discount}}{\text{MP}} \times 100\right)\% = \left(\frac{62.55}{625.50} \times 100\right)\% = 10\%$$

17.

$$\begin{aligned}\text{The MP of an article} &= ₹\ 1350\\ \text{SP} &= ₹\ 1080\\ \text{Discount \%} &= \left(\frac{\text{MP}-\text{SP}}{\text{MP}} \times 100\right)\%\\ &= \left(\frac{1350-1080}{1350} \times 100\right)\%\\ &= \left(\frac{270}{1350} \times 100\right)\% = 20\%\end{aligned}$$

18.

$$\begin{aligned}\text{discount\%} &= 10\%,\ \text{profit} = 20\%\\ \text{MP} &= ₹\ 1700\\ \text{Discount} &= \frac{\text{discount\%} \times \text{MP}}{100}\\ &= ₹\ \frac{10 \times 1700}{100} = ₹\ 170\\ \text{SP} &= \text{MP} - \text{discount}\\ &= ₹\ (1700 - 170) = ₹\ 1530\end{aligned}$$

So,

$$\begin{aligned}\text{profit\%} &= \frac{\text{SP}-\text{CP}}{\text{CP}} \times 100\\ 20 &= \frac{1530-\text{CP}}{\text{CP}} \times 100\\ \text{CP} &= 1530 \times 5 - 5\,\text{CP}\\ 6\ \text{CP} &= ₹\ 7650\\ \text{CP} &= ₹\ (7650 \div 6)\\ \text{CP} &= ₹\ 1275\end{aligned}$$

Hence, the cost price of an item is ₹ 1275.

19. discount% = 20%, profit% = 20%

$$\text{CP} = ₹\ 600,\ \text{SP} = ?,\ \text{MP} = ?$$

$$\therefore \quad \text{profit\%} = \left(\frac{\text{SP}-\text{CP}}{\text{CP}} \times 100\right)\%$$

$$20 = \frac{\text{SP}-600}{600} \times 100$$

$$120 = \text{SP} - 600$$

$$\text{SP} = ₹\ (600+120) = ₹\ 720$$

So, $$\text{Discount\%} = \frac{\text{discount}}{\text{SP}+\text{discount}} \times 100$$

$$20 = \frac{\text{discount}}{720+\text{discount}} \times 100$$

$$₹\ 720 + \text{discount} = 5\,\text{discount}$$

$$4\ \text{discount} = ₹\ 720$$

$$\text{discount} = ₹\ 720 \div 4 = ₹\ 180$$

So, $$\text{MP} = \text{SP} + \text{discount}$$

$$= ₹\ (720+180) = ₹\ 900$$

Hence, the MP of an article is ₹ 900, which is bought by him for ₹ 600.

20. $$\text{The SP of a watch} = ₹\ 728$$

$$\text{Discount\%} = 9\%$$

$$\text{MP} = ?$$

$$\text{Discount\%} = \frac{\text{discount}}{\text{SP}+\text{discount}} \times 100$$

$$9 = \frac{\text{discount}}{728+\text{discount}} \times 100$$

$$₹\ 9 \times 728 + 9\,\text{discount} = 100\,\text{discount}$$

$$91\ \text{discount} = ₹\ 6552$$

$$\text{discount} = ₹\ (6552 \div 91) = ₹\ 72$$

So, $$\text{MP} = \text{SP} + \text{discount} = ₹\ (728+72) = ₹\ 800$$

Hence, the marked price of the watch is ₹ 800.

21. Discount % $= 16\%$, gain % $= 5\%$

$$\text{Let the cost price of the article} = ₹\ 100$$

$\therefore$ $$\text{S.P} = ₹\ (100+5) = ₹\ 105 \qquad \ldots\text{(i)}$$

Again, let the marked price of the article be ₹ x.

Then, $$\text{discount} = ₹\ 16$$

$\therefore$ $$\text{Selling price} = \text{Marked price} - \text{Discount}$$

$$= ₹\left(x - \frac{16}{100}x\right) = ₹\ \frac{21}{25}x \qquad \ldots\text{(ii)}$$

From (i) and (ii)

$$\frac{21}{25}x = 105$$

$$21x = ₹\ 2625$$

$$x = ₹\ (2625 \div 21) = ₹\ 125$$

If the cost price is ₹ 100, then marked price is ₹ 125.

i.e., the dealer must mark his good 25% above the cost price.

22. Let the CP of an article be ₹ x.

Therefore, $$\text{MP} = ₹\ (x + x \text{ of } 40\%) = ₹\left(x + x \times \frac{40}{100}\right) = ₹\left(\frac{5x+2x}{5}\right) = ₹\ \frac{7}{5}x$$

So, $$\text{Discount\%} = \frac{\text{discount}}{\text{MP}} \times 100$$

$$20 = \frac{\text{discount}}{\frac{7}{5}x} \times 100$$

$$\text{discount} = \frac{7}{25}x$$

So, $$\text{SP} = \text{MP} - \text{discount} = \frac{7}{5}x - \frac{7}{25}x = \frac{35x - 7x}{25} = \frac{28}{25}x$$

Therefore, his gain %

$$= \left(\frac{SP - CP}{CP} \times 100\right)\%$$

$$= \left(\frac{\frac{28}{25}x - x}{x} \times 100\right)\% = \left(\frac{28x - 25x}{25x} \times 100\right)\% = \left(\frac{3x}{25x} \times 100\right)\% = (3 \times 4)\% = 12\%$$

Hence, 12% is his gain persent.

23. Let the CP of an article be ₹ x.

Therefore, $$\text{MP} = ₹\ (x + x \text{ of } 40\%)$$

$$= ₹\left(x + x \times \frac{40}{100}\right) = ₹\left(\frac{5x + 2x}{100}\right) = ₹\ \frac{7}{5}x$$

$$\text{discount} = \frac{7}{25}x$$

So, $$\text{Discount }\% = \frac{discount}{MP} \times 100$$

$$5 = \frac{discount}{\frac{7}{5}x} \times 100$$

$$\text{discount} = \frac{7}{100}x$$

But, $$\text{the SP of an article} = ₹\ 1064$$

∴ $$\text{SP} = \text{MP} - \text{discount}$$

$$₹\ 1064 = \frac{7}{5}x - \frac{7}{100}x$$

$$₹\ 1064 = \frac{7 \times 20x - 7x}{100} = \frac{140x - 7x}{100}$$

$$₹\ 1064 = \frac{133}{100}x$$

$$x = ₹\ \frac{1064 \times 100}{133}$$

$$x = ₹\ 800$$

Thus, $$\text{actual profit} = \text{SP} - \text{CP} = ₹\ (1064 - 800) = ₹\ 264$$

MCQs

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(d)

7

Compound Interest

Exercise 7.1

1. (a) $P =$ ₹ 10000, $R = 15\%$, $T = 2$ years

Interest for the first year = ₹ $\frac{10000\times 15\times 1}{100}$ = ₹ 1500

∴ amount at the end of first year = ₹ (10000 + 1500) = ₹ 11500

Principal for the second year = ₹ 11500

Interest for the second year = ₹ $\frac{11500\times 15\times 1}{100}$ = ₹ 1725

Hence, amount at the end of second year = ₹ (11500 + 1725)

= ₹ 13225

∴ compound interest after 2 years = ₹ (13225 − 10000)

= ₹ 3225

(b) Principal for the first year = ₹ 8000, $R = 10\%$

Interest for the first year = ₹ $\frac{8000\times 10\times 1}{100}$

= ₹ 800

∴ amount at the end of the first year = ₹ (8000 + 800)

= ₹ 8800

Interest for the second year = ₹ $\frac{8800\times 10\times 1}{100}$

= ₹ 880

∴ amount at the end of second year = ₹ (8800 + 880)

= ₹ 9680

Interest for the third year = ₹ $\frac{9680\times 10\times 1}{100}$ = ₹ 968

∴ amount at the end of third year = ₹ (9680 + 968) = ₹ 10648

Hence, compound interest = ₹ (10648 − 8000)

= ₹ 2648

(c) Principal for the first year = ₹ 5000, $R = 10\%$

Interest for the first year = ₹ $\frac{5000\times 10\times 1}{100}$

= ₹ 500

∴ amount at the end of the first year = ₹ (5000 + 500)

= ₹ 5500

Interest for the second year = ₹ $\frac{5500\times 10\times 1}{100}$

= ₹ 550

∴ amount at the end of second year = ₹ (5500 + 550)

= ₹ 6050

Hence, compound interest = ₹ (6050 − 5000)

= ₹ 1050

(d) Principal for the first year $= ₹\ 12000,\ R = 12\%$

Interest for the first year $= ₹\ \frac{12000 \times 12 \times 1}{100}$

$= ₹\ 1440$

$\therefore$ amount at the end of the first year $= ₹\ (12000 + 1440)$

$= ₹\ 13440$

Interest for the second year $= ₹\ \frac{13440 \times 12 \times 1}{100}$

$= ₹\ 1612.80$

$\therefore$ amount at the end of second year $= ₹\ (13440 + 1612.80)$

$= ₹\ 15052.80$

Hence, compound interest $= ₹\ (15052.80 - 12000) = ₹\ 3052.80$

(e) Principal for the first year $= ₹\ 15000,\ R = 20\%$

Interest for the first year $= ₹\ \frac{15000 \times 20 \times 1}{100}$

$= ₹\ 150 \times 20 = ₹\ 3000$

$\therefore$ amount at the end of first year $= ₹\ (15000 + 3000)$

$= ₹\ 18000$

Interest for the second year $= ₹\ \frac{18000 \times 20 \times 1}{100}$

$= ₹\ 180 \times 20 = ₹\ 3600$

$\therefore$ amount at the end of second year $= ₹\ (18000 + 3600)$

$= ₹\ 21600$

Interest for the third year $= ₹\ \frac{21600 \times 20 \times 1}{100}$

$= ₹\ 4320$

$\therefore$ amount at the end of third year $= ₹\ (21600 + 4320)$

$= ₹\ 25920$

Hence, compound interest $= ₹\ (25920 - 15000)$

$= ₹\ 10920$

2. $P = ₹\ 20000,\ R = 10\%$

(a) the compound interest after one year $= ₹\ \frac{20000 \times 10 \times 1}{100}$

$= ₹\ 2000$

(b) Amount at the end of first year $= ₹\ (20000 + 2000)$

$= ₹\ 22000$

Interest for the second year $= ₹\ \frac{22000 \times 10 \times 1}{100}$

$= ₹\ 2200$

$\therefore$ amount at the end of second year $= ₹\ (22000 + 2200)$

$= ₹\ 24200$

$\therefore$ the compound interest for 2 year $= ₹\ (24200 - 20000)$

$= ₹\ 4200.$

(c) The sum of money required to clean the debt at the end of 2 years is ₹ 24200.

(d) SI for two years $= ₹\ \frac{20000 \times 10 \times 2}{100}$

$= ₹\ 4000$

The difference between the compound interest and the simple interest

$= ₹\ (4200 - 4000)$

$= ₹\ 200$

3. The given, $P = ₹\ 35000$, $R = 20\%$, $T = 3$ years

Interest for the first year $= ₹\ \frac{35000 \times 20 \times 1}{100}$

$= ₹\ 7000$

$\therefore$ amount at the end of first year $= ₹\ (35000 + 7000)$

$= ₹\ 42000$

Interest for the second year $= ₹\ \frac{42000 \times 20 \times 1}{100} = ₹\ 8400$

$\therefore$ amount at the end of second year $= ₹\ (42000 + 8400)$

$= ₹\ 50400$

Interest for the third year $= ₹\ \frac{50400 \times 20 \times 1}{100} = ₹\ 10080$

$\therefore$ amount at the end of third year $= ₹\ (50400 + 10080)$

$= ₹\ 60480$

$\therefore$ the compound interest $= ₹\ (60480 - 35000)$

$= ₹\ 25480$

Hence, he will pay ₹ 25480 after 3 years.

4. $P = ₹\ 1800$, $R = 14\%$ and $T = 2$ years

Interest for the first year $= ₹\ \frac{1800 \times 14 \times 1}{100}$

$= ₹\ 252$

$\therefore$ amount at the end of first year $= ₹\ (1800 + 252) = ₹\ 2052$

Interest for the second year $= ₹\ \frac{2052 \times 14 \times 1}{100}$

$= ₹\ 287.28$

$\therefore$ amount at the end of first year $= ₹\ (2052 + 287.28)$

$= ₹\ 2339.28$

$\therefore$ the compound interest $= ₹\ (2339.28 - 1800)$

$= ₹\ 539.28$

Hence, the compound interest that Manish will get ₹ 539.28.

Exercise 7.2

1. Since, Compound interest (C.I.) $= P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$

And Amount $(A) = P + \text{C.I.}$

(a) $P = ₹\ 625$, $R = 4\%$ and $n = 2$ years

$\therefore$ C.I. $= ₹\ 625\left[\left(1 + \frac{4}{100}\right)^2 - 1\right]$

$$= ₹\ 625\left[\left(1+\frac{1}{25}\right)^2-1\right]$$

$$= ₹\ 625\left[\left(\frac{26}{25}\right)^2-1\right]$$

$$= ₹\ 625\times\left[\frac{676}{625}-1\right]$$

$$= ₹\ 625\times\left[\frac{676-625}{625}\right]$$

$$= ₹\ 625\times\frac{51}{625} = ₹\ 51$$

And $A = P + \text{C.I.} = ₹\ (625+51) = ₹\ 676$

(b) $P = ₹\ 16000$, $R = 15\%$ and $n = 3$ years

$\therefore$ $$\text{CI.} = ₹\ 16000\left[\left(1+\frac{15}{100}\right)^3-1\right]$$

$$= ₹\ 16000\left[\left(1+\frac{3}{20}\right)^3-1\right]$$

$$= ₹\ 16000\left[\left(\frac{23}{20}\right)^3-1\right]$$

$$= ₹\ 16000\times\left[\frac{12167-8000}{8000}\right]$$

$$= ₹\ 16000\times\frac{4167}{8000}$$

$$= ₹\ 2\times 4167 = ₹\ 8334$$

And $A = ₹\ (16000+8334) = ₹\ 24334$

(c) $P = ₹\ 2400$, $R = 20\%$ and $n = 3$ years

$\therefore$ $$\text{C.I.} = ₹\ 2400\left[\left(1+\frac{20}{100}\right)^3-1\right]$$

$$= ₹\ 2400\left[\left(1+\frac{1}{5}\right)^3-1\right]$$

$$= ₹\ 2400\left[\left(\frac{6}{5}\right)^3-1\right]$$

$$= ₹\ 2400\left[\frac{216-125}{125}\right]$$

$$= ₹\ 2400\times\frac{91}{125} = ₹\ 1747.20$$

And $A = P + \text{C.I.} = ₹\ (2400+1747.20)$

$= ₹\ 4147.20$

(d) $P = ₹\ 15000$, $R = 12$ paise per rupee per annum $= 12\%$ and $n = 3$ years

$$\therefore \qquad \text{CI} = ₹\ 15000\left[\left(1+\frac{12}{100}\right)^3 - 1\right]$$

$$= ₹\ 15000\left[\left(1+\frac{3}{25}\right)^3 - 1\right]$$

$$= ₹\ 15000\left[\left(\frac{28}{25}\right)^3 - 1\right]$$

$$= ₹\ 15000\left[\frac{21952-15625}{15625}\right]$$

$$= ₹\ 15000\times\frac{6327}{15623}$$

$$= ₹\ 6073.92$$

So, $A = P + \text{CI} = ₹\ (15000 + 6073.92) = ₹\ 21073.92$

(e) $P = ₹\ 5000$, $R = 20$ paise per rupee per annum $= 20\%$ and $n = 4$ years.

$$\therefore \qquad \text{CI} = ₹\ 5000\left[\left(1+\frac{20}{100}\right)^4 - 1\right]$$

$$= ₹\ 5000\left[\left(1+\frac{1}{5}\right)^4 - 1\right]$$

$$= ₹\ 5000\left[\left(\frac{6}{5}\right)^4 - 1\right]$$

$$= ₹\ 5000\left[\frac{1296}{625} - 1\right]$$

$$= ₹\ 5000\left[\frac{1296-625}{625}\right]$$

$$= ₹\ 5000\times\frac{671}{625}$$

$$= ₹\ 8\times 671 = ₹\ 5368$$

And $\qquad A = P + \text{C.I.} = ₹\ (5000 + 5368)$

$$= ₹\ 10368$$

(f) $P = ₹\ 10000$, $R = 5$ paise per rupee per annum $= 5\%$ and $n = 2$ years

$$\therefore \qquad \text{C.I.} = ₹\ 10000\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$$

$$= ₹\ 10000\left[\left(1+\frac{1}{20}\right)^2 - 1\right]$$

$$= ₹\ 10000\left[\left(\frac{21}{20}\right)^2 - 1\right]$$

$$= ₹\ 10000\left[\frac{441-400}{400}\right]$$

$$= ₹\ 10000 \times \frac{41}{400} = ₹\ 25 \times 41 = ₹\ 1025$$

And $$A = P + \text{C.I.} = ₹\ (10000 + 1025) = ₹\ 11025$$

2. The given,

$P = ₹\ 16000$, $R = 15\%$ and $n = 3$ years

We know that,

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$\therefore \quad A = ₹\ 16000\left(1 + \frac{15}{100}\right)^3$$

$$= ₹\ 16000\left(1 + \frac{3}{20}\right)^3$$

$$= ₹\ 16000\left(\frac{23}{20}\right)^3 = ₹\ 16000 \times \frac{12167}{8000}$$

$$= ₹\ 2 \times 12167 = ₹\ 24334$$

Hence, the amount that he will pay ₹ 24334 to the bank after 3 years.

3. $P = ₹\ 32000$, $R = 12\frac{1}{2}\%$ and $n = 2$ years

$$\text{Compound interest} = P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$$

$$= ₹\ 32000\left[\left(1 + \frac{\frac{25}{2}}{100}\right)^2 - 1\right]$$

$$= ₹\ 32000\left[\left(1 + \frac{1}{8}\right)^2 - 1\right]$$

$$= ₹\ 32000\left[\left(\frac{9}{8}\right)^2 - 1\right]$$

$$= ₹\ 32000\left[\frac{81 - 64}{64}\right]$$

$$= ₹\ 32000 \times \frac{17}{64} = ₹\ 8500$$

Hence, ₹ 8500 Rahim paid to Salim.

4. Here, $P = ₹\ 15625$, $R = 16\%$ and $n = 3$ years

$$\therefore \quad \text{C.I.} = ₹\ 15625\left[\left(1 + \frac{16}{100}\right)^3 - 1\right]$$

$$= ₹\ 15625\left[\left(1 + \frac{4}{25}\right)^3 - 1\right]$$

$$= ₹\ 15625\left[\left(\frac{29}{25}\right)^3 - 1\right]$$

$$= ₹\ 15625 \times \left[\frac{24389 - 15625}{15625}\right]$$

$$= ₹\ 15625 \times \frac{8764}{15625}$$

$$= ₹\ 8764$$

$\therefore \quad A = P + \text{C.I.} = ₹\ (15625 + 8764) = ₹\ 24389$

Hence, the amount that Ashok will pay ₹ 24389 after 3 years.

5. Here, $P = ₹\ 7200$, $R = 25\%$ and $n = 3$ years.

$\therefore \quad \text{Amount } = ₹\ 7200\left(1 + \frac{25}{100}\right)^3$

$$= ₹\ 7200\left(1 + \frac{1}{4}\right)^3$$

$$= ₹\ 7200 \times \frac{5 \times 5 \times 5}{4 \times 4 \times 4}$$

$$= ₹\ 7200 \times \frac{125}{64} = ₹\ 112.5 \times 125$$

$$= ₹\ 14062.50$$

6. Here, $P = ₹\ 500$, $R = 12\frac{1}{2}\% = \frac{25}{2}\%$

and $n = T = 1$ year

$\therefore \quad \text{S.I.} = ₹\ \frac{500 \times \frac{25}{2} \times 1}{100} = ₹\ \frac{5 \times 25 \times 1}{2} = ₹\ 62.50$

Also,

$\therefore \quad \text{C.I.} = ₹\ 500\left[\left(1 + \frac{\frac{25}{2}}{100}\right)^1 - 1\right]$

$$= ₹\ 500\left[\left(1 + \frac{1}{8}\right) - 1\right] = ₹\ 500 \times \frac{1}{8} = ₹\ 62.50$$

Hence, compound interest and simple interest are same as amount for one year.

Exercise 7.3

1. Here, $P = ₹\ 5000$

$R = 8\%$ per annum $= \frac{1}{2} \times 8\% = 4\%$

$n = 18$ months $= \frac{3}{2}$ years $= 2 \times \frac{3}{2} = 3$ half years.

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= ₹\ 5000\left(1+\frac{4}{100}\right)^3$$

$$= ₹\ 5000\left(1+\frac{1}{25}\right)^3$$

$$= ₹\ 5000\times\frac{26\times 26\times 26}{25\times 25\times 25}$$

$$= ₹\ \frac{8\times 26\times 26\times 26}{25} = ₹\ \frac{140608}{25} = ₹\ 5624.32$$

So, C.I. $= A - P = ₹\ (5624.32 - 5000) = ₹\ 624.32$

2. Here, $P = 72000$,

$R = 10\%$ per annum $= \frac{1}{2}\times 10\% = 5\%$ per half yearly.

$n = 1\frac{1}{2}$ years$= 2\times\frac{3}{2}$ years $= 3$ half yearly.

$$A = ₹\ 72000\left(1+\frac{5}{100}\right)^3$$

$$= ₹\ 72000\times\left(1+\frac{1}{20}\right)^3$$

$$= ₹\ 72000\times\frac{21\times 21\times 21}{20\times 20\times 20}$$

$$= ₹\ 9\times 21\times 21\times 21$$

$$= ₹\ 83349$$

So, C.I. $= A - P = ₹\ (83349 - 72000) = ₹\ 11349$

3. Here, $P = ₹\ 20000$

$R = 10\%$ per annum $= \frac{1}{2}\times 10\% = 5\%$ per half-yearly.

$n = 2$ years $= 2\times 2 = 4$ half yearly

So, C.I. $= P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$

$\therefore$ C.I. $= ₹\ 20000\left[\left(1+\frac{5}{100}\right)^4 - 1\right]$

$$= ₹\ 20000\left[\left(1+\frac{1}{20}\right)^4 - 1\right]$$

$$= ₹\ 20000\left[\left(\frac{21}{20}\right)^4 - 1\right]$$

$$= ₹\ 20000\left[\frac{194481-160000}{160000}\right]$$

$$= ₹\ 20000\times\frac{34481}{160000}$$

$$= ₹\ 4310.12$$

4. Here, $P =$ ₹ 700,

$R = 20\% = \frac{1}{2} \times 20\%$ half yearly $= 10\%$ half-yearly.

$n = 1\frac{1}{2}$ years $= \frac{3}{2} \times 2$ half yearly $= 3$ half yearly

So,
$$\text{C.I.} = ₹\ 700\left[\left(1+\frac{10}{100}\right)^3 - 1\right]$$
$$= ₹\ 700\left[\left(\frac{11}{10}\right)^3 - 1\right]$$
$$= ₹\ 700\left[\frac{1331-1000}{1000}\right]$$
$$= ₹\ 700 \times \frac{331}{1000} = ₹\ \frac{2317}{10} = ₹\ 231.70$$

Hence, the compound interest that he will pay ₹ 231.70 after one and a half years.

5. Here, $P =$ ₹ 15625,

$R = 16\%$ per annum $= \frac{1}{4} \times 16\% = 4\%$ per quarter

$n = 9$ months $= \frac{9}{12} \times 4 = 3$ quarters

$\therefore$
$$A = P\left(1+\frac{R}{100}\right)^n$$
$$= ₹\ 15625\left(1+\frac{4}{100}\right)^3$$
$$= ₹\ 15625\left(1+\frac{1}{25}\right)^3$$
$$= ₹\ 15625\left(\frac{26}{25}\right)^3$$
$$= ₹\ 15625 \times \frac{17576}{15625} = ₹\ 17576$$

Thus, the amount is ₹ 17576.

MCQs

1. (c) **2.** (a) **3.** (c) **4.** (b) **5.** (d)

8 Algebraic Expressions

Exercise 8.1

1. (a) $5x^2 y \times 8y^2 x^3 = (5 \times 8) \times (x^2 \times x^3) \times (y \times y^2)$
$$= 40 \times x^{2+3} \times y^{1+2} \quad [\because x^m \times n^n = x^{m+n}]$$
$$= 40x^5 y^3$$

(b) $$-7a^2b^3\times a^3b^4=(-7\times 1)\times(a^2\times a^3)\times(b^3\times b^4)$$
$$=-7\times a^{2+3}\times b^{3+4}\quad[\because x^m\times x^n=x^{m+n}]$$
$$=-7a^5b^7$$

(c) $$\frac{20}{7}abc\times\frac{7}{8}a^3b^2c^5=\left(\frac{20}{7}\times\frac{7}{8}\right)\times(a\times a^3)\times(b\times b^2)\times(c\times c^5)$$
$$=\frac{5}{2}\times a^{1+3}\times b^{1+2}\times c^{1+5}=\frac{5}{2}a^4b^3c^6$$

(d) $$-\frac{4}{7}a^3b\times\frac{5}{8}a^2c=\left(-\frac{4}{7}\times\frac{5}{8}\right)\times(a^3\times a^2)\times b\times c$$
$$=-\frac{5}{14}a^{3+2}\times bc=-\frac{5}{14}a^5bc$$

2. (a) $$(-4x^2y)\times(5xy^2)\times(6x^2yz)=(-4\times 5\times 6)\times(x^2\times x\times x^2)\times(y\times y^2\times y)\times z$$
$$=-120x^{2+1+2}\times y^{1+2+1}\times z=-120x^5y^4z$$

(b) $$(3xyz)\times(-2x^3y^3z^4)\times(7x^5y^5z^6)$$
$$=(-3\times 2\times 7)\times(x\times x^3\times x^5)\times(y\times y^3\times 5^5)\times(z\times z^4\times z^6)$$
$$=-42\times x^{1+3+5}\times y^{1+3+5}\times z^{1+4+6}=-42x^9y^9z^{11}$$

(c) $$\left(\frac{3}{2}x^2yz\right)\times\left(\frac{1}{5}xy^2z\right)\times(-75x^2y^2z^2)$$
$$=\left(\frac{3}{2}\times\frac{1}{5}\times(-75)\right)\times(x^2\times x\times x^2)\times(y\times y^2\times y^2)\times(z\times z\times z^2)$$
$$=-\frac{45}{2}\times x^{2+1+2}\times y^{1+2+2}\times z^{1+1+2}=-\frac{45}{2}x^5y^5z^4$$

(d) $$\left(\frac{8}{5}x^5y^5z^5\right)\times\left(\frac{14}{3}x^2y^2z^2\right)\times(-6)$$
$$=\left(-6\times\frac{8}{5}\times\frac{14}{3}\right)\times(x^5\times x^2)\times(y^5\times y^2)\times(z^5\times z^2)$$
$$=-\frac{224}{5}\times x^{5+2}\times y^{5+2}\times z^{5+2}=-\frac{224}{5}x^7y^7z^7$$

(e) $$\left(\frac{9}{4}a^2b^2c\right)\times\left(-\frac{2}{3}a^6c^4b\right)\times\left(\frac{-14}{9}a^3b^2c^5\right)$$
$$=\left(+\frac{9}{4}\times\frac{2}{3}\times\frac{14}{9}\right)\times(a^2\times a^6\times a^3)\times(b^2\times b\times b^2)\times(c\times c^4\times c^5)$$
$$=\frac{7}{3}\times a^{2+6+3}\times b^{2+1+2}\times c^{1+4+5}=\frac{7}{3}a^{11}b^5c^{10}$$

(f) $$\left(\frac{1}{21}xyz\right)\times\left(\frac{-14}{5}x^2z^2\right)\times\left(\frac{15}{9}y^3z^3\right)$$
$$=\left(-\frac{1}{21}\times\frac{14}{5}\times\frac{15}{9}\right)\times(x\times x^2)\times(y\times y^3)\times(z\times z^2\times z^3)$$
$$=-\frac{2}{9}\times x^{1+2}\times y^{1+3}\times z^{1+2+3}$$
$$=-\frac{2}{9}x^3y^4z^6$$

3. (a) $x^{7} \times x^{10} \times x^{14} \times x^{28} = x^{(7+10+14+28)}$ $[\because x^{m} \times x^{n} = x^{m+n}]$

$= x^{59}$

(b) $(x^{60}y^{40}) \times (y^{40}z^{20}) \times (z^{20}x^{60})$

$= (x^{60} \times x^{60}) \times (y^{40} \times y^{40}) \times (z^{20} \times z^{20})$

$= x^{120}y^{80}y^{40}$

(c) $x^{500} \times y^{400} \times (xyz) \times z^{0} = (x^{500} \times x^{1}) \times (y^{400} \times y^{1}) \times (z^{1} \times z^{0})$

$= x^{500+1} \times y^{400+1} \times z^{1+0}$

$= x^{501}y^{401}z^{1}$

4. $\left(\frac{1}{4}x^{5}y^{4}z^{3}\right) \times (-4x^{2}z^{2}) = \left(-\frac{1}{4} \times 4\right) \times (x^{5} \times x^{2}) \times (y^{4}) \times (z^{3} \times z^{2})$

$= -x^{7}y^{4}z^{5}$

Substituting $x = 1,\ y = 2$ and $z = 3$

$\left(\frac{1}{4}x^{5}y^{4}z^{3}\right) \times (-4x^{2}z^{2}) = -x^{7}y^{4}z^{5}$

LHS $= \left(\frac{1}{4} \times 1^{5} \times 2^{4} \times 3^{3}\right) \times (-4 \times 1^{2} \times 3^{2})$

$= \left(\frac{1}{4} \times 1 \times 16 \times 27\right) \times (-4 \times 9)$

$= (4 \times 27)(-36) = -3888$

RHS $= -1^{7} \times 2^{4} \times 3^{5} = -1 \times 16 \times 243 = -3888$

Hence, LHS = RHS

5. $(7a^{2} - ab) \times (3a^{2}b^{2}) = 3 \times 7 \times a^{2} \times a^{2} \times b^{2} - 3a \times a^{2} \times b^{2}$

$= 21a^{4}b^{2} - 3a^{3}b^{2}$

Substituting $a = 2,\ b = 1$

$(7a^{2} - ab) \times (3a^{2}b^{2}) = 21a^{4}b^{2} - 3a^{3}b^{2}$

LHS $= (7a^{2} - ab) \times (3a^{2}b^{2})$

$= (7 \times 2^{2} - 2 \times 1) \times (3 \times 2^{2} \times 1^{2})$

$= (28 - 2) \times 12 = 26 \times 12 = 312$

RHS $= 21a^{4}b^{2} - 3a^{3}b^{2}$

$= 21 \times 2^{4} \times 1^{2} - 3 \times 2^{3} \times 1^{2}$

$= 336 - 24 = 312$

Hence, LHS = RHS

6. $(25xyz) \times \left(\frac{1}{5}xyz\right)^{2} = x^{3}y^{3}z^{3}$

LHS $= (25xyz) \times \left(\frac{1}{5}xyz\right)^{2}$ $[\because x = 2,\ y = 1, y = 3]$

$= (25 \times 2 \times 1 \times 3) \times \left(\frac{1}{5} \times 2 \times 1 \times 3\right)^{2}$

$= (25 \times 6) \times \left(\frac{1}{5} \times 6\right)^{2}$

$= 150 \times \frac{36}{25} = 6 \times 36 = 216$

RHS $= x^3 y^3 z^3 = 2^3 \times 1^3 \times 3^3 = 8 \times 1 \times 27 = 216$

Hence, LHS = RHS

7. $\left(\frac{1}{8}x^2 y^2 z\right) \times \left(\frac{2}{3}xyz\right) \times (-6yz)$

$= \left[\frac{1}{8} \times \frac{2}{3} \times (-6)\right] \times (x^2 \times x) \times (y^2 \times y \times y) \times (z \times z \times z)$

$= -\frac{1}{2} \times x^3 \times y^4 \times z^3 = -\frac{1}{2}x^3 y^4 z^3$

Substituting $x = 2,\ y = 3$ and $z = 2$

$\left(\frac{1}{8}x^2 y^2 z\right) \times \left(\frac{2}{3}xyz\right) \times (-6yz) = -\frac{1}{4}x^3 y^4 z^3$

LHS $= \left(\frac{1}{8} \times 2^2 \times 3^2 \times 2\right) \times \left(\frac{2}{3} \times 2 \times 3 \times 2\right)(-6 \times 3 \times 2)$

$= \left(\frac{1}{8} \times 4 \times 9 \times 2\right) \times (8) \times (-36)$

$= -9 \times 8 \times 36 = -2592$

RHS $= -\frac{1}{2}x^3 y^4 z^3 = -\frac{1}{2} \times 2^3 \times 3^4 \times 2^3$

$= -\frac{1}{4} \times 8 \times 81 \times 8$

$= -4 \times 81 \times 8 = -2592$

Hence, LHS = RHS

8. $7x^5 \times \left(-\frac{1}{21}xy^2\right) \times 3xyz^2 = \left(-7 \times \frac{1}{21} \times 3\right) \times (x^5 \times x \times x) \times (y^2 \times y) \times z^2$

$= -1 \times x^{5+1+1} \times y^{2+1} \times z^2$

$= -x^7 y^3 z^2$

Substituting $x = 1,\ y = 2$ and $z = 3$

$= -1^7 \times 2^3 \times 3^2 = -1 \times 8 \times 9 = -72$

9. $2m^2 n^2 \times 7m^2 = 14m^2 \times m^2 \times n^2 = 14\,m^4 n^2$

Substituting $m = -\frac{1}{2}$ and $n = 4$

$= 14 \times \left(-\frac{1}{2}\right)^4 \times (4)^2 = 14 \times \frac{1}{16} \times 16 = 14$

10. (a) $\left(-\frac{17}{8}x^2 y^2\right) \times \left(-\frac{16}{51}x^2 y\right) = \left(+\frac{17}{8} \times \frac{16}{51}\right) \times (x^2 \times x^2) \times (y^2 \times y)$

$= \frac{2}{3} \times x^4 \times y^3 = \frac{2}{3}x^4 y^3$

(b) $\left(\frac{-4 \times z^2}{3}\right) \times \left(\frac{9}{16}y^2 z^2\right)\left(-\frac{14}{27}\right)x^2 y$

$$=\left(+\frac{4}{3}\times\frac{9}{16}\times\frac{14}{27}\right)\times x^2\times(y^2\times y)\times z^2\times z^2$$

$$=\frac{7}{18}\times x^2\times y^3\times z^{2+2}=\frac{7}{18}x^2y^3z^4$$

(c) $\left(-\frac{9}{33}yz\right)\times\left(\frac{-11}{66}xy^2z\right)\times\left(\frac{4}{5}x^2yz\right)$

$$=\left(+\frac{9}{33}\times\frac{11}{66}\times\frac{4}{5}\right)\times(x\times x^2)\times(y\times y^2\times y)\times(z\times z\times z)$$

$$=\frac{2}{55}\times x^3\times y^4\times z^3=\frac{2}{55}x^3y^4z^3$$

(d) $\left(\frac{5}{2}x^2y^3\right)\times(-y^2z)\times(-2x^3yz^7)\times\left(-\frac{1}{5}xyz\right)$

$$=\left(\frac{-5}{2}\times 2\times\frac{1}{5}\right)\times(x^2\times x^3\times x)\times(y^3\times y^2\times y\times y)\times(z\times z^7\times z)$$

$$=-x^6y^7z^9$$

11. $2.4\,p^7q^3\times 1.5\,p^3q=3.6\times p^{7+3}\times q^{3+1}=3.6\,p^{10}q^4$

Substituting $p=1, q=0.6$, we get

So, $3.6\times 1^{10}\times(0.6)^4=3.6\times 0.1296=0.46656$

Exercise 8.2

1. (a) $8x^3\times(5x-6y^2)=8x^3\times 5x-8x^3\times 6y^2=40x^4-48x^3y^2$

(b) $x^2(x^2y+y^2x)=x^2\times x^2y+x^2\times y^2x=x^4y+x^3y^2$

(c) $-6y\times(2xy+3y^2)=-6\times 2\times xy\times y-6\times 3\times y^2\times y=-12xy^2-18y^3$

(d) $\frac{1}{3}xy\times(4x-9y)=\frac{1}{3}xy\times 4x-\frac{1}{3}xy\times 9y=\frac{4}{3}x^2y-3xy^2$

(e) $\frac{1}{8}xy\times\left(\frac{1}{10}x^2y^2-\frac{4}{5}y\right)=\frac{1}{8}xy\times\frac{1}{10}x^2y^2-\frac{1}{8}xy\times\frac{4}{5}y$

$$=\frac{1}{80}x^3y^3-\frac{1}{2\times 5}xy^2$$

$$=\frac{1}{80}x^3y^3-\frac{1}{10}xy^2$$

(f) $\frac{7}{5}a(a^3+b^3)=\frac{7}{5}a\times a^3+\frac{7}{5}a\times b^3=\frac{7}{5}a^4+\frac{7}{5}ab^3$

(g) $-\frac{7}{4}x^2\left(\frac{2}{7}x^2+4y^2\right)=-\frac{7}{4}x^2\times\frac{2}{7}x^2-\frac{7}{4}x^2\times 4y^2=-\frac{1}{2}x^4-7x^2y^2$

(h) $\frac{1}{10}x\left(\frac{3}{5}x^2-\frac{1}{4}y^2\right)=\frac{1}{10}x\times\frac{3}{5}x^2-\frac{1}{10}x\times\frac{1}{4}y^2=\frac{3}{50}x^3-\frac{1}{40}xy^2$

(i) $\frac{2}{5}x(5x^2y-50xy^2)=\frac{2}{5}x\times 5x^2y-\frac{2}{5}x\times 50xy^2=2x^3y-20x^2y^2$

2. (a) $x^2y\times(x+z)=x^2y\times x+x^2y\times z=x^3y+x^2yz$

Verification : When $x=3$, $y=2$ and $z=1$

LHS $=x^2y\times(x+z)=3^2\times 2\times(3+1)$

$$=9\times 2\times 4=72$$

$\text{RHS} = x^3 y + x^2 yz = 3^3 \times 2 + 3^2 \times 2 \times 1$

$= 27 \times 2 + 9 \times 2 \times 1$

$= 54 + 18 = 72$

$\therefore$ LHS = RHS

(b) $xyz \times (2y^2 + 3x^2) = 2xy^3 z + 3x^3 yz$

Verification : When $x = 3,\ y = 2$ and $z = 1$

$\text{LHS} = xyz \times (2y^2 + 3x^2) = 3 \times 2 \times 1 \times (2 \times 2^2 + 3 \times 3^2)$

$= 6 \times (8 + 27) = 210$

$\text{RHS} = 2xy^3 z + 3x^3 yz = 2 \times 3 \times 2^3 \times 1 + 3 \times 3^3 \times 2 \times 1$

$= 48 + 105 = 210$

$\therefore$ LHS = RHS

(c) $6xyz \times (3x^2 - 5xz) = 6xyz \times 3x^2 - 6xyz \times 5xz = 18x^3 yz - 30x^2 yz^2$

Verification : When $x = 3,\ y = 2$ and $z = 1$

$\text{LHS} = 6xyz \times (3x^2 - 5xz)$

$= 6 \times 3 \times 2 \times 1 \times (3 \times 3^2 - 5 \times 3 \times 1) = 36 \times (27 - 15) = 36 \times 12 = 432$

$\text{RHS} = 18x^3 yz - 30x^2 yz^2$

$= 18 \times 3^3 \times 2 \times 1 - 30 \times 3^2 \times 2 \times 1^2$

$= 18 \times 27 \times 2 - 30 \times 9 \times 2 \times 1$

$= 972 - 540 = 432$

$\therefore$ LHS = RHS

(d) $5x^3 y \times (y - z^2) = 5x^3 y \times y - 5x^3 y \times z^2 = 5x^3 y^2 - 5x^3 yz^2$

Verification : When $x = 3,\ y = 2$ and $z = 1$

$\text{LHS} = 5x^3 y(y - z^2) = 5 \times 3^3 \times 2(2 - 1^2) = 5 \times 27 \times 2 \times 1 = 270$

$\text{RHS} = 5x^3 y^2 - 5x^3 yz^2 = 5 \times 3^3 \times 2^2 - 5 \times 3^3 \times 2 \times 1^2 = 540 - 270 = 270$

$\therefore$ LHS = RHS

3. (a) $(2x + 5y) \times (4x + 7y) = 2x \times (4x + 7y) + 5y \times (4x + 7y)$

$= 2x \times 4x + 2x \times 7y + 5y \times 4x + 5y \times 7y$

$= 8x^2 + 14xy + 20xy + 35y^2$

$= 8x^2 + 34xy + 35y^2$

(b) $(5x^2 + 4y)(5x - 8y) = 5x^2 \times (5x - 8y) + 4y(5x - 8y)$

$= 5x^2 \times 5x - 5x^2 \times 8y + 4y \times 5x - 4y \times 8y$

$= 25x^3 - 40x^2 y + 20xy - 32y^2$

$= 25x^3 + 20xy - 40x^2 y - 32y^2$

(c) $\left(\frac{5}{4}xy + \frac{7}{4}y^2 z\right) \times (3x - 6yz)$

$= \frac{5}{4}xy \times (3x - 6yz) + \frac{7}{4}y^2 z(3x - 6yz)$

$= \frac{5}{4}xy \times 3x - \frac{5}{4}xy \times 6yz + \frac{7}{4}y^2 z \times 3x - \frac{7}{4}y^2 z \times 6yz$

$= \frac{15}{4}x^2 y - \frac{15}{2}xy^2 z + \frac{21}{4}xy^2 z - \frac{21}{2}y^3 z^2$

$$= \frac{15}{4}x^2y + \left(\frac{-15}{2} + \frac{21}{4}\right)xy^2z - \frac{21}{2}y^3z^2$$

$$= \frac{15}{4}x^2y - \frac{9}{4}xy^2z - \frac{21}{2}y^3z^2$$

(d) $(2+3yz)\times(x^2y^2+4yz) = 2\times(x^2y^2+4yz) + 3yz\,(x^2y^2+4yz)$

$$= 2x^2y^2 + 8yz + 3x^2y^3z + 12y^2z^2$$

(e) $(3x^2y^2+2y^2z^2)\times(8z^2x^2-9x^2y^2)$

$$= 3x^2y^2\times(8z^2x^2-9x^2y^2) + 2y^2z^2\,(8z^2x^2-9x^2y^2)$$

$$= 8\times3x^2y^2\times z^2\times x^2 - 3\times9x^2y^2 \times x^2y^2$$

$$+ 2y^2z^2\times8z^2x^2 - 2y^2z^2\times9x^2y^2$$

$$= 24x^4y^2z^2 - 27x^4y^4 + 16x^2y^2z^4 - 18x^2y^4z^2$$

$$= 24x^4y^2z^2 + 16x^2y^2z^4 - 27x^4y^4 - 18x^2y^4z^2$$

4. (a) $5x^2 + 4x(4x-7y) - 10xy = 5x^2 + 4x\times4x - 4x\times7y - 10xy$

$$= 5x^2 + 16x^2 - 28xy - 10xy$$

$$= 21x^2 - 38xy$$

(b) $x^2y + 3x^2y\times(2x+7y) - 5x^2y^2 = x^2y + 3x^2y\times2x + 3x^2y\times7y - 5x^2y^2$

$$= x^2y + 6x^3y + 21x^2y^2 - 5x^2y^2$$

$$= x^2y + 6x^3y + 16x^2y^2$$

(c) $2xy\times3y^2z + 5x^2y\times(8y-3x) = 6xy^3z + 5x^2y\times8y - 5x^2y\times3x$

$$= 6xy^3z + 40x^2y^2 - 15x^3y$$

(d) $\frac{1}{7}(8x^2+20y^2)(x^2-y^2)$

$$= \frac{1}{7}8x^2(x^2-y^2) + \frac{1}{7}\times20y^2(x^2-y^2)$$

$$= \frac{1}{7}8x^2\times x^2 - \frac{1}{7}8x^2\times y^2 + \frac{1}{7}\times20y^2\times x^2 - \frac{1}{7}20y^2\times y^2$$

$$= \frac{8}{7}x^4 - \frac{8}{7}x^2y^2 + \frac{20}{7}x^2y^2 - \frac{20}{7}y^4$$

$$= \frac{8}{7}x^4 + \left(-\frac{8}{7} + \frac{20}{7}\right)x^2y^2 - \frac{20}{7}y^4$$

$$= \frac{8}{7}x^4 + \frac{12}{7}x^2y^2 - \frac{20}{7}y^4$$

5. (a) $(3xy+5y^2)\times(5y^2+4x^2)$

$$= 3xy(5y^2+4x^2) + 5y^2(5y^2+4x^2)$$

$$= 3xy\times5y^2 + 3xy\times4x^2 + 5y^2\times5y^2 + 5y^2\times4x^2$$

$$= 15xy^3 + 12x^3y + 25y^4 + 20x^2y^2$$

Verification : When $x=1,\ y=2$

LHS $= (3xy+5y^2)\times(5y^2+4x^2)$

$$= (3\times1\times2+5\times2^2)\times(5\times2^2+4\times1^2)$$

$$= (6+20) \times (20+4) = 26 \times 24 = 624$$

$$\text{RHS} = 15xy^3 + 12x^3 y + 25y^4 + 20x^2 y^2$$

$$= 15 \times 1 \times 2^3 + 12 \times 1^3 \times 2 + 25 \times 2^4 + 20 \times 1^2 \times 2^2$$

$$= 15 \times 8 + 12 \times 2 + 25 \times 16 + 20 \times 4$$

$$= 120 + 24 + 400 + 80$$

$$= 624$$

So, $\text{LHS} = \text{RHS}$

(b) $(10y + 7x^2) \times \left(\frac{5}{2}x^3 + xy\right)$

$$= 10y \times \left(\frac{5}{2}x^3 + xy\right) + 7x^2 \times \left(\frac{5}{2}x^3 + xy\right)$$

$$= 10y \times \frac{5}{2}x^3 + 10y \times xy + 7x^2 \times \frac{5}{2}x^3 + 7x^2 \times xy$$

$$= 25x^3 y + 10xy^2 + \frac{35}{2}x^5 + 7x^3 y$$

$$= 32x^3 y + 10xy^2 + \frac{35}{2}x^5$$

Verification : When $x = 1,\ y = 2$

$$\text{LHS} = (10y + 7x^2) \times \left(\frac{5}{2}x^3 + xy\right)$$

$$= (10 \times 2 + 7 \times 1^2) \times \left(\frac{5}{2} \times 1^3 + 1 \times 2\right)$$

$$= (20 + 7) \times \left(\frac{5}{2} + 2\right)$$

$$= 27 \times \frac{9}{2} = \frac{243}{3}$$

$\therefore$ $\text{LHS} = \text{RHS}$

(c) $\left(5x^2 + \frac{1}{10}y^2\right) \times (3y - 4x^2)$

$$= 5x^2 \times (3y - 4x^2) + \frac{1}{10}y^2 (3y - 4x^2)$$

$$= 5x^2 \times 3y - 5x^2 \times 4x^2 + \frac{1}{10}y^2 \times 3y - \frac{1}{10}y^2 \times 4x^2$$

$$= 15x^2 y - 20x^4 + \frac{3}{10}y^3 - \frac{2}{5}x^2 y^2 = 15x^2 y + \frac{3}{10}y^3 - 20x^4 - \frac{2}{5}x^2 y^2$$

Verification : When $x = 1,\ y = 2$

$$\text{LHS} \quad = \left(5x^2 + \frac{1}{10}y^2\right) \times (3y - 4x^2)$$

$$= \left(5 \times 1^2 + \frac{1}{10} \times 2^2\right) \times (3 \times 2 - 4 \times 1^2)$$

$$= \left(5 + \frac{4}{10}\right) \times (6 - 4)$$

$$= \left(\frac{25+2}{5}\right) \times 2 = \frac{27}{5} \times 2 = \frac{54}{5}$$

(d) $\left(6x+\frac{1}{3}y^3\right)\times(x^3-y^3)=6x\times(x^3-y^3)+\frac{1}{3}y^3(x^3-y^3)$

$$=6x\times x^3-6x\times y^3+\frac{1}{3}y^3\times x^3-\frac{1}{3}y^3\times y^3$$

$$=6x^4-6xy^3+\frac{1}{3}x^3y^3-\frac{1}{3}y^6$$

Verification : When $x=1,\ y=2$

$$\left(6x+\frac{1}{3}y^3\right)\times(x^3-y^3)=\left(6\times1+\frac{1}{3}\times2^3\right)\times(1^3-2^3)$$

$$=\left(6+\frac{8}{3}\right)\times(1-8)$$

$$=\left(\frac{18+8}{3}\right)(-7)=-\frac{26\times7}{3}=\frac{-182}{3}$$

6. (a) $9x^2\times(3x+4)\times(9x+1)=(9x^2\times3x+9x^2\times4)\times(9x+1)$

$$=(27x^3+36x^2)\times(9x+1)$$

$$=27x^3\times(9x+1)+36x^2\times(9x+1)$$

$$=27x^3\times9x+27x^3+36x^2\times9x+36x^2$$

$$=243x^4+27x^3+324x^3+36x^2$$

$$=243x^4+351x^3+36x^2$$

(b) $\frac{1}{5}x\times(10x-4)\times(7-14x)=\left(\frac{1}{5}x\times10x-\frac{1}{5}x\times4\right)\times(7-14x)$

$$=\left(2x^2-\frac{4}{5}x\right)\times(7-14x)$$

$$=2x^2\times7-2x^2\times14x-\frac{4}{5}x\times7+\frac{4}{5}x\times14x$$

$$=14x^2-28x^3-\frac{28}{5}x+\frac{56}{5}x^2$$

$$=\left(14+\frac{56}{5}\right)x^2-28x^3-\frac{28}{5}x$$

$$=\left(\frac{70+56}{5}\right)x^2-28x^3-\frac{28}{5}x$$

$$=\frac{125}{5}x^2-\frac{28}{5}x-28x^3$$

or $\qquad -28x^3+\frac{125}{5}x^2-\frac{28}{5}x$

(c) $(x^3+6)\times4x\times(7x-2)\times5x^2$

$$=[4x\times x^3+6\times4x]\times[7x\times5x^2-2\times5x^2]$$

$$=(4x^4+24x)(35x^3-10x^2)$$

$$=4x^4\times35x^3-4x^4\times10x^2+24x\times35x^3-24x\times10x^2$$

$$=140x^7-40x^6+840x^4-240x^3$$

(d) $(4x+3)\times2x\times(-x+1)=(4x\times2x+3\times2x)\times(-x+1)$

$$=(8x^2+6x)\times(-x+1)$$

$$= -8x^2 \times x + 8x^2 \times 1 - 6x \times x + 6x$$
$$= -8x^3 + 8x^2 - 6x^2 + 6x$$
$$= 6x + 2x^2 - 8x^3$$

7. (a) $(a+b)(a^2 - ab + b^2) = a \times a^2 - a \times ab + ab^2 + a^2b - ab^2 + b \times b^2$
$$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$
$$= a^3 + b^3$$

(b) $(4a+b)(7a+6b+2) = 4a \times 7a + 4a \times 6b + 4a \times 2 + 7a \times b + 6b \times b + 2 \times b$
$$= 28a^2 + 24ab + 8a + 7ab + 6b^2 + 2b$$
$$= 28a^2 + 8a + 31ab + 6b^2 + 2b$$

(c) $(a^3 + b^3)(a+b+c) = a^3 \times a + a^3 \times b + a^3 \times c + a \times b^3 + b^3 \times b + b^3c$
$$= a^4 + a^3b + a^3c + ab^3 + b^4 + b^3c$$
$$= a^4 + b^4 + a^3b + a^3c + b^3c + b^3a$$

(d) $(a^2 + b^2 + c^2) \times (b^2 - c^2)$
$$= a^2 \times b^2 - a^2 \times c^2 + b^2 \times b^2 - b^2 \times c^2 + b^2 \times c^2 - c^2 \times c^2$$
$$= a^2b^2 - a^2c^2 + b^4 - b^2c^2 + b^2c^2 - c^4$$
$$= a^2b^2 - a^2c^2 + b^4 - c^4$$
$$= b^4 - c^4 + a^2b^2 - a^2c^2$$

8. (a) $(5x+y)(1+10x+3y) - 10x(x+y)$
$$= 5x + 5x \times 10x + 5x \times 3y + y + 10xy + 3y^2 - 10xy - 10x^2$$
$$= 5x + 50x^2 + 15xy + y + 10xy + 3y^2 - 10xy - 10x^2$$
$$= 5x + 40x^2 + 15xy + y + 3y^2$$

(b) $(2y + 5x - 6)(x-1) - (y - 2x + 5)(x+7)$
$$= (2y \times x + 5x \times x - 6x - 2y - 5x + 6) - (xy - 2x^2 + 5x + 7y - 14x + 35)$$
$$= 2xy + 5x^2 - 6x - 2y - 5x + 6 - xy + 2x^2 - 5x - 7y + 14x - 35$$
$$= xy + 7x^2 - 9y - 2x - 29$$

(c) $p^2 + (4p - q)(4p + q + q^2) = p^2 + 16p^2 + 4pq + 4pq^2 - 4pq - q^2 - q^3$
$$= 17p^2 - q^3 - q^2 + 4pq^2$$

(d) $a^2b(a - b^2) + ab^2(7ab - 3b^2) - a^2b(8 - 3b)$
$$= a^2b \times a - a^2b \times b^2 + 7ab \times ab^2 - 3b^2 \times ab^2 - a^2b \times 8 + a^2b \times 3b$$
$$= a^3b - a^2b^3 + 7a^2b^3 - 3ab^4 - 8a^2b + 3a^2b^2$$
$$= a^3b + 3a^2b^2 - 3ab^4 - 8a^2b + 6a^2b^3$$

Exercise 8.3

1. (a) $\left(3a^2 + \dfrac{1}{3a^2}\right)^2$ $\quad [\because (x \pm y)^2 = x^2 \pm 2xy + y^2]$
$$= (3a^2)^2 + \left(\frac{1}{3a^2}\right)^2 + 2 \times 3a^2 \times \frac{1}{3a^2}$$
$$= 9a^4 + \frac{1}{9a^4} + 2$$

(b) $$(6x-y^2)^2=(6x)^2+(y^2)^2-2\times 6x\times y^2$$
$$=36x^2+y^4-12xy^2$$
or $$=36x^2-12xy^2+y^4$$

(c) $$\left(\frac{1}{5}x^2+\frac{1}{3}y^2\right)^2=\left(\frac{1}{5}x^2\right)^2+\left(\frac{1}{3}y^2\right)^2+2\times\frac{1}{5}x^2\times\frac{1}{3}y^2$$
$$=\frac{1}{25}x^4+\frac{1}{9}y^4+\frac{2}{15}x^2y^2$$
$$=\frac{1}{25}x^4+\frac{2}{15}x^2y^2+\frac{1}{9}y^4$$

(d) $$\left(15+\frac{1}{3}\right)^2=15^2+\left(\frac{1}{3}\right)^2+2\times 15\times\frac{1}{3}=225+\frac{1}{9}+10=235+\frac{1}{9}=235.11$$

(e) $$\left(\frac{1}{2}xy-z^2\right)^2=\left(\frac{1}{2}xy\right)^2+(z^2)^2-2\times\frac{1}{2}xy\times z^2$$
$$=\frac{1}{4}x^2y^2+z^4-xyz^2$$
$$=\frac{1}{4}x^2y^2-xyz^2+z^4$$

(f) $$\left(5x^2+\frac{1}{x}\right)\left(5x^2-\frac{1}{x}\right)$$
$$=(5x^2)^2-\left(\frac{1}{x}\right)^2\quad[\because (a+b)(a-b)=a^2-b^2]$$
$$=25x^4-\frac{1}{x^2}$$

2. (a) $$(x+5)(x+5)=x(x+5)+5(x+5)=x^2+5x+5x+25=x^2+10x+25$$

(b) $$(x^2+25)(x^2+25)=x^2(x^2+25)+25(x^2+25)$$
$$=x^4+25x^2+25x^2+625$$
$$=x^4+50x^2+625$$

(c) $$(xy+z)(xy+z)=xy\times(xy+z)+z(xy+z)$$
$$=x^2y^2+xyz+xyz+z^2$$
$$=xy^2+2xyz+z^2$$

(d) $$(3x-4y)(3x-4y)=3x(3x-4y)-4y(3x-4y)$$
$$=9x^2-12xy-12xy+16y^2$$
$$=9x^2-24xy+16y^2$$

(e) $$(5x^2y^2-3y^2)(5x^2y^2-3y^2)$$
$$=5x^2y^2(5x^2y^2-3y^2)-3y^2(5x^2y^2-3y^2)$$
$$=5x^2y^2\times 5x^2y^2-5x^2y^2\times 3y^2-3y^2\times 5x^2y^2+9y^4$$
$$=25x^4y^4-15x^2y^4-15x^2y^4+9y^4$$
$$=25x^4y^4-30x^2y^4+9y^4$$

(f) $$\left(\frac{1}{5}ab-cd\right)\left(\frac{1}{5}ab-cd\right)$$

$$=\frac{1}{5}ab\left(\frac{1}{5}ab-cd\right)-cd\left(\frac{1}{5}ab-cd\right)$$

$$=\frac{1}{5}ab\times\frac{1}{5}ab-\frac{1}{5}ab\times cd-cd\times\frac{1}{5}ab+cd\times cd$$

$$=\frac{1}{25}a^2b^2-\frac{1}{5}abcd-\frac{1}{5}abcd+c^2d^2$$

$$=\frac{1}{25}a^2b^2-\frac{2}{5}abcd+c^2d^2$$

(g) $(4yz+7x^2)(4yz-7x^2)=4yz(4yz-7x^2)+7x^2(4yz-7x^2)$

$$=16y^2z^2-28x^2yz+28x^2yz-49x^4$$

$$=16y^2z^2-49x^4$$

(h) $(2a^3-b^3)(2a^3+b^3)=(2a^3)^2-(b^3)^2=4a^6-b^6$

(i) $(yz^2+x^2)(yz^2-x^2)=(yz^2)^2-(x^2)^2=y^2z^4-x^4$

3. (a) $(7x+4)^2-(7x-4)^2=(7x+4+7x-4)(7x+4-7x+4)$

$$=(14x)\times(8)=112x$$

(b) $\left(\frac{4}{7}x+1\right)^2+\left(\frac{4}{7}x-1\right)^2=\left(\frac{4}{7}x\right)^2+1^2+2\times\frac{4}{7}x\times1+\left(\frac{4}{7}x\right)^2+1^2-2\times\frac{4}{7}x\times1$

$$=\frac{16}{49}x^2+1+\frac{8}{7}x+\frac{16}{49}x^2+1-\frac{8}{7}x$$

$$=\frac{32}{49}x^2+2$$

(c) $\left(\frac{2x}{5}-\frac{y}{2}\right)^2-\left(\frac{2x}{5}+\frac{y}{2}\right)^2$

$$=\left[\left(\frac{2x}{5}\right)^2+\left(\frac{y}{2}\right)^2-2\times\frac{2x}{5}\times\frac{y}{2}\right]-\left[\left(\frac{2x}{5}\right)^2+\left(\frac{y}{2}\right)^2+2\times\frac{2x}{5}\times\frac{y}{2}\right]$$

$$=\left[\frac{4}{25}x^2+\frac{y^2}{4}-\frac{2xy}{5}\right]-\left[\frac{4x^2}{25}+\frac{y^2}{4}+\frac{2xy}{5}\right]$$

$$=\frac{4}{25}x^2+\frac{y^2}{4}-\frac{2xy}{5}-\frac{4}{25}x^2-\frac{y^2}{4}-\frac{2xy}{5}=-\frac{4xy}{5}$$

4. Since, $a^2-b^2=(a+b)(a-b)$

(a) $(100)^2-(5)^2=(100+5)(100-5)=105\times95=9975$

(b) $61^2-39^2=(61+39)(61-39)=2200$

(c) $315^2-205^2=(315+205)(315-205)=520\times110=57200$

(d) $750^2-650^2=(750+650)(750-650)=1400\times100=140000$

5. Since, $(a+b)^2=a^2+2ab+b^2$

And $(a-b)^2=a^2-2ab+b^2$

(a) $(104)^2=(100+4)^2$

$$=(100)^2+2\times100\times4+(4)^2=10000+800+16=10816$$

(b) $(1006)^2=(1000+6)^2$

$$=1000^2+2\times1000\times6+6^2=1000000+12000+36=1012036$$

(c) $(996)^2=(1000-4)^2$

$$=(1000)^2-2\times1000\times4+(4)^2$$

$$=1000000-8000+16=992000+16=992016$$

(d) $34^2=(30+4)^2=(30)^2+2\times30\times4+(4)^2=900+240+16=1156$

(e) $(95)^2=(100-5)^2$

$$=(100)^2-2\times100\times5+(5)^2$$

$$=10000-1000+25=9000+25=9025$$

(f) $100.4\times99.6=(100+0.4)\ (100-0.4)$

$$=(100)^2-(0.4)^2=10000-0.16\quad=9999.84$$

(g) $54\times46=(50+4)\times(50-4)=(50)^2-(4)^2=2500-16=2484$

(h) $(205)^2=(200+5)^2$

$$=(200)^2+2\times200\times5+5^2=40000+2000+25=42025$$

6. (a) $\dfrac{97\times97-3\times3}{(97-3)}\qquad\because a^2-b^2=(a+b)(a-b)$

$$=\frac{(97)^2-3^2}{(97-3)}=\frac{(97-3)(97+3)}{(97-3)}=(97+3)=100$$

(b) $\dfrac{8.12\times8.12-0.12\times0.12}{8.24}$

$$=\frac{(8.12)^2-(0.12)^2}{8.24}=\frac{(8.12+0.12)(8.12-0.12)}{8.24}=\frac{8.24\times8}{8.24}=8$$

7. (a)

$$15x=40\times40-25\times25$$

$$15x=(40)^2-(25)^2$$

$$15x=(40+25)(40-25)$$

$$15x=65\times15$$

$$x=\frac{65\times15}{15}$$

$$x=65$$

(b)

$$21x=78\times78-57\times57$$

$$21x=(78)^2-(57)^2$$

$$21x=(78+57)(78-57)$$

$$21x=135\times21$$

$$x=\frac{135\times21}{21}$$

$$x=135$$

8. (a)

$$24x=75^2-63^2$$

$$24x=(75+63)(75-63)$$

$$24x=138\times12$$

$$x=\frac{138\times12}{24}$$

$$x=69$$

(b) $$60x = 53^2 - 37^2$$
$$60x = (53+37)(53-37)$$
$$60x = 90 \times 16$$
$$x = \frac{90 \times 16}{60}$$
$$x = 24$$

(c) $$12x = (45)^2 - (27)^2$$
$$12x = (45+27)(45-27)$$
$$12x = 72 \times 18$$
$$x = \frac{72 \times 18}{12}$$
$$x = 108$$

9. $(7x+4y)(7x-4y)(49x^2+16y^2) = [(7x)^2-(4y)^2][49x^2+16y^2]$
$$= [(7x)^2-(4y)^2][(7x)^2+(4y)^2]$$
$$= \{(7x)^2\}^2 - \{(4y)^2\}^2$$
$$= (7x)^4 - (4y)^4 = 2401x^4 - 256y^4$$

Substituting : When $x=1, y=2$

$\text{LHS} = (7x+4y)(7x-4y)(49x^2+16y^2)$
$= (7\times 1+4\times 2)(7\times 1-4\times 2)(49\times 1^2+16\times 2^2)$
$= (7+8)(7-4)(49+64)$
$= 15\times 3\times 3136$
$= 141120$

$\text{RHS} = 2401x^4 - 256y^4$
$= 2401\times 1 - 256\times 2^4 = 2401-4096 = -1695$

$\therefore$ $\text{LHS} = \text{RHS}$

10. (a) $(x+3)(x-3)(x^2-9)$
$= (x^2-3^2)(x^2-9)$ [Using $(a+b)(a-b) = a^2-b^2$]
$= (x^2-9)(x^2-9)$
$= (x^2-9)^2$
$= x^4 - 2\times x^2\times 9 + 9^2$ [Using $(a-b)^2 = a^2-2ab+b^2$]
$= x^4 - 18x^2 + 81$

(b) $(4x-1)(4x+1)(16x^2+1)$
$= [(4x)^2-(1)^2](16x^2+1)$ [Using $(a+b)(a-b) = a^2-b^2$]
$= (16x^2-1)(16x^2+1)$
$= (16x^2)^2-(1)^2$
$= 256x^4 - 1$

(c) $(ax+b)(ax-b)(a^2x^2+b^2)$
$= [(ax)^2-(b)^2](a^2x^2+b^2)$ [Using $(a+b)(a-b) = a^2-b^2$]
$= (a^2x^2-b^2)(a^2x^2+b^2)$

$$= (a^2x^2)^2 - (b^2)^2$$

$$= a^4x^4 - b^4$$

(d) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right)$

$$= \left[(x)^2 - \left(\frac{1}{x}\right)^2\right]\left(x^2-\frac{1}{x^2}\right) \quad \text{[Using } (a+b)(a-b) = a^2 - b^2\text{]}$$

$$= \left(x^2-\frac{1}{x^2}\right)\left(x^2-\frac{1}{x^2}\right)$$

$$= \left(x^2-\frac{1}{x^2}\right)^2$$

$$(x^2)^2 - 2x^2 \times \frac{1}{x^2} + \left(\frac{1}{x^2}\right) \text{[Using } (a-b)^2 = a^2 - 2ab + b^2\text{]}$$

$$= x^4 - 2 + \frac{1}{x^4}$$

11. We have, $\left(x+\frac{1}{x}\right) = 11$

Squaring both sides, we get

$$\left(x+\frac{1}{x}\right)^2 = 11^2$$

$$x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = 121$$

$$x^2 + \frac{1}{x^2} + 2 = 121$$

$$x^2 + \frac{1}{x^2} = 121 - 2$$

$$x^2 + \frac{1}{x^2} = 119$$

Thus, $x^2 + \frac{1}{x^2} = 119$

12. We have, $\left(x+\frac{1}{x}\right) = 5$

Squaring both sides, we get

$$\left(x+\frac{1}{x}\right)^2 = (5)^2$$

$$x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = 25$$

$$x^2 + 2 + \frac{1}{x^2} = 25$$

$$x^2 + \frac{1}{x^2} = 25 - 2$$

$$x^2 + \frac{1}{x^2} = 23 \qquad \ldots(i)$$

Again squaring both sides of equation (i), we get

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 23^2$$

$$x^4 + 2 \times x^2 \times \frac{1}{x^2} + \frac{1}{x^4} = 529$$

$$x^4 + 2 + \frac{1}{x^4} = 529$$

$$x^4 + \frac{1}{x^4} = 529 - 2$$

$$x^4 + \frac{1}{x^4} = 527$$

13. We have,

$$\left(x - \frac{1}{x}\right) = 7$$

Squaring both sides, we get

$$\left(x - \frac{1}{x}\right)^2 = 7^2$$

$$x^2 - 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = 49$$

$$x^2 - 2 + \frac{1}{x^2} = 49$$

$$x^2 + \frac{1}{x^2} = 49 + 2$$

$$x^2 + \frac{1}{x^2} = 51 \qquad ...(i)$$

Again squaring both sides of equation (i), we get

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 51^2$$

$$(x^2)^2 + 2x^2 \times \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 2601$$

$$x^4 + 2 + \frac{1}{x^4} = 2601$$

$$x^4 + \frac{1}{x^4} = 2601 - 2$$

$$x^4 + \frac{1}{x^4} = 2599$$

14. We have,

$$\left(x-\frac{1}{x}\right) = 3$$

Squaring both sides, we get

$$\left(x-\frac{1}{x}\right)^2 = 3^2$$

$$x^2 - 2\times x\times\frac{1}{x}+\frac{1}{x^2} = 9$$

$$x^2 - 2+\frac{1}{x^2} = 9$$

$$x^2 + \frac{1}{x^2} = 9+2$$

$$x^2 + \frac{1}{x^2} = 11 \qquad \ldots\text{(i)}$$

Again squaring both sides of equation (i), we get

$$\left(x^2+\frac{1}{x^2}\right)^2 = 11^2$$

$$(x^2)^2 + 2x^2\times\frac{1}{x^2}+\left(\frac{1}{x^2}\right)^2 = 121$$

$$x^4 + 2+\frac{1}{x^4} = 121$$

$$x^4 + \frac{1}{x^4} = 119$$

Exercise 8.4

1. (a) The greatest common factor of xy, x^2y and xyz is xy.

   (b) The greatest common factor of 5 and 15 is 5 and the greatest common factor of x^2y and x^2y^2 is x^2y.

   Hence, the greatest common factor of $5x^2y$ and $15x^2y^2$ is $5x^2y$.

   (c) The greatest common factor 24, 6 and 4 is 2 and the greatest common factor of xyz, x^2y and y^2x is xy.

   Hence, the greatest common factor of $24xyz$, $6x^2y$ and $4y^2x$ is $2xy$.

   (d) The greatest common factor 16, 12 and 24 is 4 and the greatest common factor of x^3y^4, x^2y^2 and xyz is xy.

   Hence, the greatest common factor of $16x^3y^4, 12x^2y^2$ and $24xyz$ is $4xy$.

2. (a) $7x^2 + 21xz = 7x(x+3z)$

(b) $8x^3 - x^2 = x^2(8x-1)$

(c) $20x^2 + 2xy + 6y^2z = 2(10x^2 + xy + 3y^2z)$

(d) $15x^5 + 45x^2y^3 - 25x^2y^2 = 5x^2(3x^2 + 9y^3 - 5y^2)$

3. (a) $p^2 + p = p(p+1)$

(b) $pqr - pq = pq(r-1)$

(c) $-7x^5 - 14x^3y - 28x^2y^2 = -7x^2(x^3 + 2xy + 4y^2)$

(d) $21m + 12mn + 15m^2 = 3m(7 + 4n + 5m)$

(e) $3x^2y + 12xy^2 = 3xy(x + 4y)$

(f) $-14a^3b^2 - 7a^2b^3 + 21 = -7(2a^3b^2 + a^2b^2 - 3)$

(g) $6ab^2 + 15a^2b^3 + 21a^3b^2 = 3ab^2(2 + 5ab + 7a^2)$

4. (a) $xp^2 + yq^2 + yp^2 + xq^2 = (xp^2 + xq^2) + (yq^2 + yp^2)$

$= x(p^2 + q^2) + y(p^2 + q^2)$

$= (p^2 + q^2)(x + y) = (x + y)(p^2 + q^2)$

(b) $al + bl + am + bm = l(a+b) + m(a+b) = (a+b)(l+m)$

(c) $x^2 - xy + 4x - 4y = x(x-y) + 4(x-y) = (x-y)(x+4)$

(d) $3 + 2xy + 6x + y = (3 + 6x) + (2xy + y)$

$= 3(1+2x) + y(1+2x) = (3+y)(2x+1)$

(e) $6xy - y^2 + 12xz - 2yz = y(6x - y) + 2z(6x - y) = (y + 2z)(6x - y)$

(f) $50 + x^2y + 5y + 10x^2 = (5y + 50) + (x^2y + 10x^2)$

$= 5(y+10) + x^2(y+10) = (y+10)(5+x^2)$

5. (a) $6ab + 18b + 36 = 6(ab + 3b + 6)$

(b) $x - 6x^2 - 3x^3 = x(1 - 6x - 3x^2)$

(c) $15x^2 + 5xy + 25x^2y^2 = 5x(3x + y + 5xy^2)$

6. (a) $36x^2 - 4y^2$ $\quad\because (a+b)(a-b) = a^2 - b^2$

$= (6x)^2 - (2y)^2 = (6x + 2y)(6x - 2y)$

(b) $4x^2 - 9y^2 = (2x)^2 - (3y)^2 = (2x + 3y)(2x - 3y)$

(c) $81x^2 - 1 = (9x)^2 - (1)^2 = (9x+1)(9x-1)$

(d) $25x^2y^2 - z^2 = (5xy)^2 - (z)^2 = (5xy + z)(5xy - z)$

(e) $81 - 25z^2 = (9)^2 - (5z)^2 = (9 + 5z)(9 - 5z)$

(f) $a^2b^2 - 49 = (ab)^2 - (7)^2 = (ab+7)(ab-7)$

7. (a) $x^2 + 12xy + 36y^2$ $\quad[\because x^2 + 2xy + y^2 = (x+y)^2]$

$= (x)^2 + 2 \times x \times 6y + (6y)^2 = (x + 6y)^2$

(b) $9x^2 + 24x + 16 = (3x)^2 + 2 \times 3x \times 4 + (4)^2 = (3x+4)^2$

(c) $64x^2 + 16x + 1 = (8x)^2 + 2 \times 8x \times 1 + (1)^2 = (8x+1)^2$

(d) $m^2 + 2 + \frac{1}{m^2} = (m)^2 + 2 \times m \times \frac{1}{m} + \left(\frac{1}{m}\right)^2 = \left(m + \frac{1}{m}\right)^2$

(e) $25a^2 + 60a + 36 = (5a)^2 + 2 \times 5a \times 6 + (6)^2 = (5a+6)^2$

(f) $25m^2 + 4n^2 + 20mn = (5m)^2 + 2 \times 5m \times 2n + (2n)^2 = (5m + 2n)^2$

8. (a) $9 - 6x + x^2$ $\quad[\because x^2 - 2xy + y^2 = (x-y)^2]$

$= (3)^2 - 2 \times 3 \times x + (x)^2 = (3-x)^2$

(b) $9a^2 - 6a + 1 = (3a)^2 - 2 \times 3a \times 1 + (1)^2 = (3a-1)^2$

(c) $4x^2 - 16x + 16 = (2x)^2 - 2 \times 2x \times 4 + (4)^2 = (2x-4)^2$

(d) $a^2x^2 - 2axb + b^2 = (ax)^2 - 2 \times ax \times b + (b)^2 = (ax - b)^2$

(e) $\frac{1}{9}x^2 - \frac{2}{3}x + 1 = \left(\frac{1}{3}x\right)^2 - 2 \times \frac{1}{3}x \times 1 + (1)^2 = \left(\frac{1}{3}x - 1\right)^2$

(f) $4x^2y^2 - 4xyz + z^2 = (2xy)^2 - 2 \times 2xy \times z + (z)^2 = (2xy - z)^2$

9. (a) $x^4 - 81$ $\qquad [\because a^2 - b^2 = (a+b)(a-b)]$

$= (x^2)^2 - (9)^2$

$= (x^2+9)(x^2-9) = (x^2+9)[(x)^2 - (3)^2] = (x^2+9)(x+3)(x-3)$

(b) $(xy)^4 - z^4 = [(xy)^2]^2 - (z^2)^2$

$= (x^2y^2 + z^2)(x^2y^2 - z^2) = (x^2y^2 + z^2)(xy + z)(xy - z)$

(c) $(9x^2 + 42x + 49)^2 - z^4 = [(3x)^2 + 2 \times 3x \times 7 + (7)^2]^2 - (z^2)^2$

$= [(3x+7)^2]^2 - (z^2)^2$

$= [(3x+7)^2 + z^2][(3x+7)^2 - z^2]$

$= (9x^2 + 42x + 49 + z)[(3x+7+z)(3x+7-z)]$

$= (3x+7-z)(3x+7+z)(9x^2 + 42x + 49 + z^2)$

10. (a) $49a^2 - 25b^6$ $\qquad [\because a^2 - b^2 = (a+b)(a-b)]$

$= (7a)^2 - (5b^3)^2 = (7a + 5b^3)(7a - 5b^3)$

(b) $25m^{36} - 49n^{14} = (5m^{18})^2 - (7n^7)^2 = (5m^{18} + 7n^7)(5m^{18} - 7n^7)$

(c) $x^2y^4z^6 - x^{16} = (xy^2z^3)^3 - (x^8)^2$

$= (xy^2z^3 + x^8)(xy^2z^3 - x^8)$

$= x^2(y^2z^3 + x^7)(y^2z^3 - x^7)$

11. (a) $6xy + x^2 + 9y^2 - 9b^2$ $\qquad [\because (a+b)(a-b) = a^2 - b^2]$

$= (x^2 + 6xy + 9y^2) - 9b^2$

$= (x+3z)^2 - 9b^2 = (x+3y)^2 - (3b)^2 = (x+3y+3b)(x+3y-3b)$

(b) $4x^2 - 12xy + 9y^2 - 49 = [(2x)^2 - 2 \times 2x \times 3y + (3y)^2] - (7)^2$

$= [(2x-3y)^2 - (7)^2] = (2x-3y+7)(2x-3y-7)$

(c) $x^2 - (a^2 + 10ab + 25b^2) = x^2 - (a+5b)^2$

$= (x-a-5b)(x+a+5b)$

Exercise 8.5

1. (a) The simplified form of $-7x^2 + 5x - x^3 + 5 - 3x^2$ is $-x^3 - 10x^2 + 5x + 5$.

(b) The simplified form of $8x^4 + 3x^2 - 15 + 2x^3 - 2x^4 + 7$ is $6x^4 + 2x^3 + 3x^2 - 8$.

(c) The simplified form of $-8x^5 + 2x^4 - 3x^2 + 8x^3 + 16x^5 - 7x^4 - 3x^3 + 12$ is $8x^5 - 5x^4 + 5x^3 - 3x^2 + 12$.

2. (a) $2 + 12x - 11x^2 - 4x^3 + 15x^4$ is in ascending order.

(b) $-3x + 5x^2 + 7x^3 + 3x^4$ is in ascending order.

(c) $4 - 6x + 10x^2 + 5x^3 - 10x^4$ is in ascending order.

(d) $-3b + 3ab + 2a^2b - 2a^4 + 8a^2b^3$ is in ascending order.

3. (a) $9x^4+8x^2-4x+7$ is in descending order.

(b) $11x^4+5x^3-10x^2+7x-7$ is in descending order.

(c) $12\,m^4n^2+9m^5+18mn^3-7m$ is in descending order.

(d) $18xy^5+13x^2y^3-7y+15$ is in descending order.

4. (a) The degree of $4x^2+3xy-y^2$ is 2.

(b) The degree of $x^2+x^2y+x^5y$ is 6.

(c) The degree of $6x-9$ is 1.

(d) The degree of 25 is 0.

(e) The degree of $8x^3y^3z^3-4$ is 9.

(f) The degree of $x^4+7x^2y+y^2$ is 4.

5. (a) $35x^2y^2\div 5y=\dfrac{35x^2y^2}{5y}=7x^2y$

(b) $-18x^3y^3z\div 3xyz=\dfrac{-18x^3y^3z}{3xyz}=-6x^2y^2$

(c) $36a^4\div(-9a^3)=\dfrac{36a^4}{-9a^3}=-4a$

(d) $-x^5y^9\div(-xy^4)=\dfrac{-x^5y^9}{-xy^4}=x^4y^5$

(e) $21a^2b^7c^4\div 3a^2b^2c^2=\dfrac{21a^2b^7c^4}{3a^2b^2c^2}=7b^5c^2$

(f) $-40x^3y^3z^3\div(-5x)=\dfrac{-40x^3y^3z^3}{-5x}=8x^2y^3z^3$

6. (a) $(2t-6t^2)\div(-t)=\dfrac{2t}{-t}-\dfrac{6t^2}{-t}=-2+6t$

(b) $(10a^3+2ab)\div 2a=\dfrac{10a^3}{2a}+\dfrac{2ab}{2a}=5a^2+b$

(c) $(5c^2-c)\div c=\dfrac{5c^2}{c}-\dfrac{c}{c}=5c-1$

(d) $(x^4y-2x^3y^2+x^5y^5)\div x^2y=\dfrac{x^4y}{x^2y}-\dfrac{2x^3y^2}{x^2y}+\dfrac{x^5y^5}{x^2y}$

$=x^2-2xy+x^3y^4$

(e) $(x^3+x^2)\div x^2=\dfrac{x^3}{x^2}+\dfrac{x^2}{x^2}=x+1$

(f) $\left(-3x^2+\dfrac{9}{4}xy-6xz\right)\div(-3x)=\dfrac{-3x^2}{-3x}+\dfrac{9xy}{4\times(-3x)}-\dfrac{6xz}{(-3x)}$

$=x-\dfrac{3}{4}y+2z$

(g) $(-21x^5a^3-14x^4a^4)\div(7x^3a^2)=\dfrac{-21x^5a^3}{7x^3a^2}-\dfrac{14x^4a^4}{7x^3a^2}=-3x^2a-2xa^2$

(h) $(-10a^3b^2+15a^2b^3-5a^2b^2)\div\left(-\frac{5}{2}a^2b^2\right)$

$$=\frac{-10a^3b^2}{-\frac{5}{2}a^2b^2}+\frac{15a^2b^2}{-\frac{5}{2}a^2b^2}+\frac{5a^2b^2}{\frac{5}{2}a^2b^2}=4a-6b+2$$

Exercise 8.6

1. (a)

$$\begin{array}{l} \qquad\qquad m+24 \\ (m-3)\overline{)\,m^2+21m-72} \\ \qquad\;\; m^2-3m \\ \qquad\;\underline{-\quad +\qquad\quad} \\ \qquad\qquad 24m-72 \\ \qquad\qquad 24m-72 \\ \qquad\quad \underline{-\qquad +\quad} \\ \qquad\qquad\quad \times \end{array}$$

Hence, $(m^2+21m+72)\div(m-3)=(m+24)$

(b)

$$\begin{array}{l} \qquad\qquad x+8 \\ (x-3)\overline{)\,x^2+5x-24} \\ \qquad\;\; x^2-3x \\ \qquad\underline{-\quad +\qquad\quad} \\ \qquad\qquad 8x-24 \\ \qquad\qquad 8x-24 \\ \qquad\quad \underline{-\quad +\quad} \\ \qquad\qquad\quad \times \end{array}$$

Hence, $(x^2+5x-24)\div(x-3)=(x+8)$

(c)

$$\begin{array}{l} \qquad\qquad 3x-5 \\ (2x-7)\overline{)\,6x^2-31x+35} \\ \qquad\;\; 6x^2-21x \\ \qquad\;\underline{-\quad +\qquad\quad} \\ \qquad\qquad -10x+35 \\ \qquad\qquad -10x+35 \\ \qquad\quad \underline{+\qquad -\quad} \\ \qquad\qquad\quad \times \end{array}$$

Hence, $(6x^2-31x+35)\div(2x-7)=3x-5$

(d)

$$\begin{array}{l} \qquad\qquad x+5 \\ (2x+1)\overline{)\,2x^2+11x+5} \\ \qquad\;\; 2x^2+x \\ \qquad\;\underline{-\quad -\qquad\quad} \\ \qquad\qquad 10x+5 \\ \qquad\qquad 10x+5 \\ \qquad\quad \underline{-\quad -\quad} \\ \qquad\qquad\quad \times \end{array}$$

Hence, $(2x^2+11x+5)\div(2x+1)=(x+5)$

(e)
$$\begin{array}{l}
\qquad\qquad\quad 2x^2-4x+3 \\
x^2-1\overline{\big)\,2x^4-4x^3+x^2+4x-3} \\
\qquad\quad 2x^4-2x^2 \\
\qquad\underline{\;-\qquad\;+\qquad\quad} \\
\qquad\qquad\; -4x^3+3x^2+4x \\
\qquad\qquad\; -4x^3+4x \\
\qquad\qquad\underline{\;+\qquad\;-\qquad\qquad\;} \\
\qquad\qquad\qquad\quad 3x^2-3 \\
\qquad\qquad\qquad\quad 3x^2-3 \\
\qquad\qquad\qquad\underline{\;-\qquad\;+\;\;} \\
\qquad\qquad\qquad\qquad\;\times
\end{array}$$

Hence, $(2x^4-4x^3+x^2+4x-3)\div(x^2-1)=2x^2-4x+3$

(f)
$$\begin{array}{l}
\qquad\quad x+16 \\
x-5\overline{\big)\,x^2+11x-80} \\
\qquad x^2-5x \\
\quad\underline{\;-\quad\;+\qquad} \\
\qquad\quad 16x-80 \\
\qquad\quad 16x-80 \\
\qquad\underline{\;-\qquad+\quad} \\
\qquad\qquad\;\times
\end{array}$$

Hence, $(x^2+11x-80)\div(x-5)=x+16$

2. (a)
$$\begin{array}{l}
\qquad\quad x^2-5x \\
x+5\overline{\big)\,x^3+4x-3} \\
\qquad x^3+5x^2 \\
\quad\underline{\;-\quad\;-\qquad\qquad} \\
\qquad\quad -5x^2+4x \\
\qquad\quad -5x^2-25x \\
\qquad\underline{\;+\qquad\;+\quad\;} \\
\qquad\qquad\qquad 29x-3 \qquad\qquad\qquad \text{R}
\end{array}$$

Thus, quotient is x^2-5x.

Check : Dividend = (divisor × quotient) + remainder

$\text{RHS}=(x+5)(x^2-5x)+(29x-3)$

$\quad\;\;=(x^3-5x^2+5x^2-25x)+(29x-3)$

$\quad\;\;=x^3-25x+29x-3=x^3+4x-3=\text{LHS}$

(b)
$$\begin{array}{l}
\qquad\qquad\; z^2+3 \\
4z^2-5\overline{\big)\,4z^4+7z^2+15} \\
\qquad\quad 4z^4-5z^2 \\
\qquad\underline{\;-\qquad+\qquad\qquad\quad}
\end{array}$$

$$\begin{array}{rl} & 12z^2+15 \\ & 12z^2-15 \\ & \underline{- \qquad +} \\ & \qquad 30 \qquad R \end{array}$$

Thus, quotient is z^2+3.

Check : Dividend = (divisor × quotient) + remainder

$\text{RHS} = (4z^2-5)(z^2+3)+30$

$= 4z^2 \times z^2 - 5z^2 + 4z^2 \times 3 - 5 \times 3$

$= 4z^4 - 5z^2 + 12z^2 - 15$

$= 4z^4 + 7z^2 - 15$

$= \text{LHS}$

(c) $$\begin{array}{rl} & -3y+4 \\ (2y-3) & \overline{\big)\,-6y^2+17y-12} \\ & -6y^2+9y \\ & \underline{+ \qquad -} \\ & \qquad 8y-12 \\ & \qquad 8y-12 \\ & \qquad \underline{- \quad +} \\ & \qquad\quad \times \end{array}$$

Thus, quotient is $(-3y+4)$.

Check : Dividend = (divisor × quotient) + remainder

$\text{RHS} = (2y-3)(-3y+4)+0 = -2y \times 3y + 4 \times 2y + 3 \times 3y + 4 \times 3$

$= -6y^2 + 8y + 9y + 12$

$= -6y^2 + 17y + 12 = \text{LHS}$

(d) $$\begin{array}{rl} & 3m^2+2m-1 \\ (3m-1) & \overline{\big)\,9m^3+3m^2-5m+7} \\ & 9m^3-3m^2 \\ & \underline{- \qquad +} \\ & \qquad 6m^2-5m \\ & \qquad 6m^2-2m \\ & \qquad \underline{- \qquad +} \\ & \qquad\qquad -3m+7 \\ & \qquad\qquad -3m+1 \\ & \qquad\qquad \underline{+ \quad -} \\ & \qquad\qquad\quad 6 \qquad R \end{array}$$

Check : Dividend = (divisor × quotient) + remainder

$\text{RHS} = (3m-1)(3m^2+2m-1)+6$

$= 9m^3 + 6m^2 - 3m - 3m^2 - 2m + 1 + 6 = 9m^3 + 3m^2 - 5m + 7 = \text{LHS}$

(e) $$\begin{array}{rl} & 3x+4 \\ (x^2+1) & \overline{\big)\,3x^3+4x^2+x+7} \\ & 3x^3+3x \\ & \underline{- \qquad -} \end{array}$$

$$\begin{array}{l} 4x^2 - 2x + 7 \\ 4x^2 \quad + 4 \\ \underline{- \qquad\quad -} \\ \qquad -2x + 3 \qquad \text{R} \end{array}$$

Thus, quotient is $3x + 4$.

Check : Dividend = (divisor × quotient) + remainder

$\text{RHS} = (x^2 + 1)(3x + 4) + (-2x + 3)$

$= 3x \times x^2 + 4x^2 + 3x + 4 - 2x + 3$

$= 3x^3 + 4x^2 + x + 7 = \text{LHS}$

(f)
$$\begin{array}{l} \qquad\qquad\quad 2x^2 + 1 \\ x^2 + 4x + 3 \overline{)\,2x^4 + 8x^3 + 7x^2 + 4x + 3} \\ \qquad\qquad\; 2x^4 + 8x^3 + 6x^2 \\ \qquad\qquad \underline{- \qquad - \qquad -\qquad\qquad\qquad} \\ \qquad\qquad\qquad\qquad\quad x^2 + 4x + 3 \\ \qquad\qquad\qquad\qquad\quad x^2 + 4x + 3 \\ \qquad\qquad\qquad\qquad\; \underline{- \qquad - \qquad -} \\ \qquad\qquad\qquad\qquad\qquad\quad 0 \qquad\qquad \text{R} \end{array}$$

Thus, quotient is $2x^2 + 1$.

Check : Dividend = (divisor × quotient) + remainder

$\text{RHS} = (x^2 + 4x + 3)(2x^2 + 1) + 0$

$= 2x^2 \times x^2 + x^2 + 4x \times 2x^2 + 4x + 3 \times 2x^2 + 3$

$= 2x^4 + 8x^3 + x^2 + 6x^2 + 4x + 3$

$= 2x^4 + 8x^3 + 7x^2 + 4x + 3 = \text{LHS}$

3. (a)
$$\begin{array}{l} \qquad\qquad\quad x^2 + 4 \\ (2x^2 - 9)\overline{)\,2x^4 - x^2 - 36} \\ \qquad\qquad 2x^4 - 9x^2 \\ \qquad\quad \underline{- \qquad +\qquad\quad} \\ \qquad\qquad\qquad 8x^2 - 36 \\ \qquad\qquad\qquad 8x^2 - 36 \\ \qquad\qquad\quad \underline{- \qquad +} \\ \qquad\qquad\qquad\quad 0 \qquad\quad \text{R} \end{array}$$

Thus, quotient is $(x^2 + 4)$ and remainder $= 0$.

(b)
$$\begin{array}{l} \qquad\qquad\; 2x + 3 \\ (x^2 + 4)\overline{)\,2x^3 + 3x^2 + 7x + 15} \\ \qquad\quad 2x^3 \qquad\quad + 8x \\ \qquad \underline{- \qquad\qquad\quad -\qquad} \\ \qquad\qquad\quad 3x^2 - x + 15 \\ \qquad\qquad\quad 3x^2 \quad\;\; + 12 \\ \qquad\qquad \underline{- \qquad\quad -\quad} \\ \qquad\qquad\qquad\quad -(x + 3) \qquad \text{R} \end{array}$$

Thus, quotient is $(2x + 3)$ and remainder is $-(x + 3)$.

(c)
$$\begin{array}{rl} & 3x-3 \\ (2x+3) & \overline{)6x^2+3x-10} \\ & 6x^2+9x \\ & \underline{-\qquad -} \\ & \quad -6x-10 \\ & \quad -6x-9 \\ & \quad \underline{+\qquad +} \\ & \qquad -1 \qquad R \end{array}$$

Thus, quotient is $(3x-3)$ and remainder is -1.

(d)
$$\begin{array}{rl} & 2y^3+2y^2+y+1 \\ (y+3) & \overline{)2y^4+7y^2+8y^3+4y+3} \\ & 2y^4+6y^3 \\ & \underline{-\qquad -} \\ & \quad +2y^3+7y^2 \\ & \quad +2y^3+6y^2 \\ & \quad \underline{-\qquad -} \\ & \qquad y^2+4y \\ & \qquad y^2+3y \\ & \qquad \underline{-\quad -} \\ & \qquad y+3 \\ & \qquad y+3 \\ & \qquad \underline{-\quad -} \\ & \qquad 0 \qquad R \end{array}$$

Thus, quotient is $(2y^3+2y^2+y+1)$ and remainder is 0.

4. (a)
$$\begin{array}{rl} & (x-12) \\ (x-12) & \overline{)x^2-24x+114} \\ & x^2-12x \\ & \underline{-\qquad +} \\ & \quad -12x+114 \\ & \quad -12x+114 \\ & \quad \underline{+\qquad -} \\ & \qquad 0 \qquad R \end{array}$$

Yes, the first polynomial is a factor of the second polynomial because remainder is zero.

(b)
$$\begin{array}{rl} & x+7 \\ (x+8) & \overline{)x^2+15x+56} \\ & x^2+8x \\ & \underline{-\qquad -} \\ & \quad 7x+56 \\ & \quad 7x+56 \\ & \quad \underline{-\qquad -} \\ & \qquad 0 \qquad R \end{array}$$

Yes, the first polynomial is a factor of the second polynomial because remainder is zero..

(c)
$$\begin{array}{r l}
 & a+4 \\
(a-7) & \overline{)a^2-3a-28} \\
 & a^2-7a \\
 & \underline{-\quad +} \\
 & \quad 4a-28 \\
 & \quad 4a-28 \\
 & \quad \underline{-\quad +} \\
 & \qquad 0 \qquad R
\end{array}$$

Yes, the first polynomial is a factor of the second polynomial because remainder is zero.

(d)
$$\begin{array}{r l}
 & 3y^3-5y-1 \\
(2y^2-6) & \overline{)6y^5-28y^3+2y^2+30y-9} \\
 & 6y^5-18y^3 \\
 & \underline{-\qquad +} \\
 & \quad -10y^3+2y^2+30y \\
 & \quad -10y^3 \qquad +30y \\
 & \quad \underline{+ \qquad\qquad -} \\
 & \qquad\quad 2y^2-9 \\
 & \qquad\quad 2y^2-6 \\
 & \qquad\quad \underline{-\quad +} \\
 & \qquad\qquad -3
\end{array}$$

No, the first polynomial is not a factor of the second polynomial.
R

MCQs

1. (b) **2.** (a) **3.** (c) **4.** (d) **5.** (d)

9 Linear Equations in One Variable

Exercise 9.1

1. (a)
$$\begin{aligned}
3x-5 &= 20-2x \\
3x+2x-5 &= 20 \text{ [Transposing } 2x \text{ from RHS to LHS]} \\
5x-5 &= 20 \\
5x &= 20+5 \text{ [Transposing 5 from LHS to RHS]} \\
5x &= 25 \\
\frac{5x}{5} &= \frac{25}{5} \text{ [Dividing both sides by 5]} \\
x &= 5
\end{aligned}$$

Check : Substituting $x = 5$ in the given equation

LHS $= 3x-5 = 3\times 5-5 = 15-5 = 10$

RHS $= 20-2x = 20-2\times 5 = 20-10 = 10$

$\therefore$ LHS = RHS, Hence, the solution is correct

(b)
$$\begin{aligned}
21x-2 &= 5x-4 \\
21x-5x &= 2-4 \text{ [Transposing } 5x \text{ and 2 LHS to RHS and} \\
&\qquad \text{LHS to LHS respectively.]} \\
16x &= -2
\end{aligned}$$

$$x = -\frac{2}{16} = -\frac{1}{8}$$

Check : Substituting $x = -\frac{1}{8}$ in the given equation, we get

$\text{LHS} = 21x - 2 = 21 \times \left(-\frac{1}{8}\right) - 2 = -\frac{21}{8} - 2 = \frac{-21-16}{8} = \frac{-37}{8}$

$\text{RHS} = 5x - 4 = 5 \times \left(-\frac{1}{8}\right) - 4 = -\frac{5}{8} - 4 = \frac{-37}{8}$

$\therefore$ LHS = RHS

Hence, the solution is correct.

(c)
$$\begin{aligned} 19x + 10 &= 17x + 22 \text{ [Transposing } 17x \text{ and 10 from RHS to} \\ &\quad \text{LHS and LHS to RHS respectively]} \\ 19x - 17x &= 22 - 10 \\ 2x &= 12 \\ \frac{2x}{2} &= \frac{12}{2} \text{ [Dividing both sides by 2]} \end{aligned}$$

Check : Substituting $x = 6$ in the given equation, we get

$\text{LHS} = 19x + 10 = 19 \times 6 + 10 = 114 + 10 = 124$

$\text{RHS} = 17x + 22 = 17 \times 6 + 22 = 102 + 22 = 124$

$\therefore$ LHS = RHS

Hence, the solution is correct

(d)
$$\begin{aligned} 0.35y - 2 &= 0.1y + 1 \\ 0.35\ y - 0.1\ y - 2 &= 1 \text{ [Transposing } 0.1\ y \text{ from RHS to LHS]} \\ 0.25y - 2 &= 1 \text{ [Transposing 2 from LHS to RHS]} \\ 0.25y &= 2 + 1 \\ \frac{0.25y}{0.25} &= \frac{3}{0.25} \text{ [Dividing both sides by 0.25]} \\ y &= 12 \end{aligned}$$

Check : Substituting $y = 12$ in the given equation, we get

$\text{LHS} = 0.35\ y - 2 = 0.35 \times 12 - 2 = 4.\ 2 - 2 = 2.2$

$\text{RHS} = 0.1y + 1 = 0.1 \times 12 + 1 = 1.2 + 1 = 2.2$

$\therefore$ LHS = RHS

Hence, the solution is correct.

2. (a)
$$\begin{aligned} \frac{5x+2}{x+4} &= 3 \\ 5x + 2 &= 3(x+4) \\ 5x + 2 &= 3x + 12 \\ 5x - 3x &= 12 - 2 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Check : Substituting $x = 5$ in the given equation, we get

$\text{LHS} = \frac{5x+2}{x+4} = \frac{5 \times 5 + 2}{5+4} = \frac{25+2}{9} = \frac{27}{9} = 3 = \text{RHS}$

Hence, the solution is correct.

(b)
$$8x + \frac{3}{4} = 3x + 7$$

$$8x-3x=7-\frac{3}{4}$$

$$5x=\frac{7\times4-3}{4}$$

$$5x=\frac{28-3}{4}$$

$$5x=\frac{25}{4} \qquad x=\frac{25}{5\times4}=\frac{5}{4}$$

Check : Substituting $x=\frac{5}{4}$ in the given equation, we get.

LHS $=8\times\frac{5}{4}+\frac{3}{4}=2\times5+\frac{3}{4}=10+\frac{3}{4}=\frac{40+3}{4}=\frac{43}{4}$

RHS $=3x+7=3\times\frac{5}{4}+7=\frac{15+28}{4}=\frac{43}{4}$

$\therefore$ LHS $=$ RHS

(c) $$7x+\frac{3}{4}=\frac{3}{2}x+7$$

$$7x-\frac{3}{2}x=7-\frac{3}{4}$$

$$\frac{7x\times2-3x}{2}=\frac{7\times4-3}{4}$$

$$\frac{14x-3x}{2}=\frac{28-3}{4}$$

$$\frac{11x}{2}=\frac{25}{4}$$

$$22x=25 \qquad x=\frac{25}{22}$$

Check : Substituting $x=\frac{25}{22}$ in the given equation, we get

LHS $=7x+\frac{3}{4}=7\times\frac{25}{22}+\frac{3}{4}=\frac{175}{22}+\frac{3}{4}=\frac{383}{44}$

RHS $=\frac{3}{2}x+7=\frac{3}{2}\times\frac{25}{22}+7=\frac{75}{44}+7=\frac{383}{44}$

$\therefore$ LHS $=$ RHS

Hence, the solution is correct.

(d) $$\frac{1}{2}x+7x-6=7x+\frac{1}{4}$$

$$\left(\frac{1}{2}+7-7\right)x=6+\frac{1}{4}$$

$$\frac{1}{2}x=\frac{24+1}{4}$$

$$x=\frac{25}{2}$$

Check : Substituting $x=\frac{25}{2}$ in the given equation, we get

$$\text{LHS} = \frac{1}{2}x + 7x - 6 = \frac{1}{2} \times \frac{25}{2} + 7 \times \frac{25}{2} - 6 = \frac{25}{4} + \frac{175}{2} - 6 = \frac{25+350-24}{4} = \frac{351}{4}$$

$$\text{RHS} = 7x + \frac{1}{4} = 7 \times \frac{25}{2} + \frac{1}{4} = \frac{175}{2} + \frac{1}{4} = \frac{350+1}{4} = \frac{351}{4}$$

$\therefore$ LHS $=$ RHS

Hence, the solution is correct.

(e)

$$\begin{aligned} \frac{5x-1}{2} - \frac{5x-2}{3} &= 1 \\ \frac{3(5x-1)-2(5x-2)}{6} &= 1 \\ 15x-3-10x+4 &= 6 \\ 15x-10x+1 &= 6 \\ 5x &= 6-1 \\ x &= \frac{5}{5} = 1 \end{aligned}$$

Check : Substituting $x = 1$ in the given equation, we get

$$\begin{aligned} \text{LHS} &= \frac{5x-1}{2} - \frac{5x-2}{3} \\ &= \frac{5 \times 1 - 1}{2} - \frac{5 \times 1 - 2}{3} = \frac{5-1}{2} - \frac{5-2}{3} = \frac{4}{2} - \frac{3}{3} = 2 - 1 = 1 = \text{RHS} \end{aligned}$$

$\therefore$ LHS $=$ RHS

Hence, the solution is correct.

(f)

$$\begin{aligned} \frac{5}{4}(7x-3) - \left(4x - \frac{1-x}{2}\right) &= x + \frac{3}{2} \\ \frac{5}{4} \times 7x - \frac{5}{4} \times 3 - 4x + \frac{1-x}{2} - x &= \frac{3}{2} \\ \frac{35}{4}x - \frac{15}{4} - 4x + \frac{1-x}{2} - x &= \frac{3}{2} \\ \frac{35x - 16x + 2 - 2x - 4x}{4} &= \frac{3}{2} + \frac{15}{4} \\ \frac{35x-22x}{4} + \frac{1}{2} &= \frac{6+15}{4} \\ \frac{13}{4}x &= \frac{21}{4} - \frac{1}{2} \\ \frac{13}{4}x &= \frac{21-2}{4} \\ \frac{13}{4}x &= \frac{19}{4} \end{aligned} \qquad x = \frac{19}{13}$$

Check : Substituting $x = \frac{19}{13}$ in the given equation, we get

$$\text{LHS} = \frac{5}{4}\left(7 \times \frac{19}{13} - 3\right) - \left(4 \times \frac{19}{13} - \frac{1 - \frac{19}{13}}{2}\right)$$

$$= \frac{5}{4}\left(\frac{133}{13} - 3\right) - \left(\frac{76}{13} - \frac{13-19}{26}\right)$$

$$= \frac{5}{4}\left(\frac{133-19}{13}\right) - \left(\frac{152+6}{26}\right)$$

$$= \frac{5}{4} \times \frac{94}{13} - \frac{158}{26} = \frac{470}{52} - \frac{158}{26} = \frac{470-316}{52} = \frac{154}{52} = \frac{77}{26}$$

$\text{RHS} = x + \frac{3}{2} = \frac{19}{13} + \frac{3}{2} = \frac{77}{26}$, $\therefore$ LHS = RHS

Hence, the solution is correct.

(g)

$$\begin{aligned} \frac{2}{5}(x-10) &= 2x+3 \\ \frac{2x}{5} - \frac{2 \times 10}{5} &= 2x+3 \\ \frac{2x}{5} - 4 &= 2x+3 \\ \frac{2x}{5} - 2x &= 4+3 \\ \frac{2x-10x}{5} &= 7 \\ \frac{-8x}{5} &= 7 \\ x &= -\frac{35}{8} \end{aligned}$$

Check : Substituting $x = -\frac{35}{8}$ in the given equation, we get

$$\text{LHS} = \frac{2}{5}(x-10) = \frac{2}{5}\left(-\frac{35}{8} - 10\right) = \frac{2}{5}\left(\frac{-35-80}{8}\right) = \frac{-2 \times 115}{5 \times 8} = -\frac{23}{4}$$

$$\text{RHS} = 2x + 3 = 2 \times \left(\frac{-35}{8}\right) + 3 = \frac{-35}{4} + 3 = -\frac{23}{4}$$

$\therefore$ LHS = RHS

Hence, the solution is correct.

(h)

$$\begin{aligned} \left(\frac{6m-2}{4}\right) + \frac{1}{3}(2m-1) &= 4\,m \\ \frac{3(6m-2) + 4(2m-1)}{12} &= 4m \\ 18m - 6 + 8m - 4 &= 48m \\ 26m - 10 &= 48m \\ \\ 22m &= -10 \\ m &= -\frac{10}{22} = -\frac{5}{11} \end{aligned}$$

Check : Substituting $m = -\frac{5}{11}$ in the given equation, we get

$$\text{LHS} = \left(\frac{6m-2}{4}\right) + \frac{1}{3}(2m-1) = \left(\frac{6\times\left(-\frac{5}{11}\right)-2}{4}\right) + \frac{1}{3}\left(2\times\left(-\frac{5}{11}\right)-1\right)$$

$$= \left(\frac{\frac{-30}{11}-2}{4}\right) + \frac{1}{3}\left(\frac{-10}{11}-1\right)$$

$$= \left(\frac{-30-22}{44}\right) + \frac{1}{3}\left(\frac{-10-11}{11}\right)$$

$$= \frac{-52}{44} + \frac{-21}{33} = \frac{-156+(-84)}{132} = \frac{-240}{132} = \frac{-20}{11}$$

$$\text{RHS} = 4m = 4\times\left(\frac{-5}{11}\right) = \frac{-20}{11}$$

$\therefore$ LHS = RHS

Hence, the solution is correct.

3. (a)

$$\begin{aligned}
6(3u-1)+3(2u+3) &= 1-7u \\
6\times 3u-6+3\times 2u+3\times 3 &= 1-7u \\
18u-6+6u+9 &= 1-7u \\
18u+6u+7u &= 1-3 \\
31u &= -2 \\
u &= -\frac{2}{31}
\end{aligned}$$

(b)

$$\begin{aligned}
2-3(3x+1) &= 2(7-6x) \\
2-3\times 3x-3 &= 2\times 7-2\times 6x \\
2-9x-3 &= 14-12x \\
12x-9x &= 14+3-2 \\
3x &= 15 \\
x &= 15\div 3 \\
x &= 5
\end{aligned}$$

(c)

$$\begin{aligned}
2(3x+2)+\frac{1}{4} &= 5x-\frac{2}{3} \\
2\times 3x+2\times 2+\frac{1}{4} &= 5x-\frac{2}{3} \\
6x+4+\frac{1}{4} &= 5x-\frac{2}{3} \\
6x-5x &= -\frac{2}{3}-\frac{1}{4}-4 \\
x &= \frac{-8-3-48}{12} \\
x &= -\frac{59}{12}
\end{aligned}$$

(d)

$$\frac{9x-1}{3x+2} = \frac{3x-5}{x+6}$$

$$\begin{aligned}
(9x-1)(x+6) &= (3x-5)(3x+2) \\
&\text{[By cross multiplication]} \\
9x\times x+9x\times 6-x-6 &= 3x\times 3x+3x\times 2-5\times 3x-5\times 2 \\
9x^2+54x-x-6 &= 9x^2+6x-15x-10 \\
53x+9x &= 6-10 \\
62x &= -4 \\
x &= \frac{-2}{31}
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{5}{7+2x} &= \frac{3}{2x+1} \\
5(2x+1) &= 3(7+2x)\,\text{[By cross multiplication]} \\
5\times 2x+5 &= 3\times 7+3\times 2x \\
10x+5 &= 21+6x \\
10x-6x &= 21-5 \\
4x &= 16 \\
x &= 16\div 4 \\
x &= 4
\end{aligned}$$

(f)

$$\begin{aligned}
\frac{8y-1}{2y+1} &= \frac{4y-5}{y+2} \\
(8y-1)(y+2) &= (4y-5)(2y+1) \\
&\text{[By cross multiplication]} \\
8y\times y+8y\times 2-y-2 &= 4y\times 2y+4y-5\times 2y-5 \\
8y^2+16y-2-y &= 8y^2+4y-10y-5 \\
16y-y-4y+10y &= 2-5 \\
21y &= -3 \\
y &= \frac{-3}{21}
\end{aligned}$$

(g)

$$\begin{aligned}
\frac{9x-(3+4x)}{2x-(7-5x)} &= \frac{6}{7} \\
9x\times 7-7(3+4x) &= 2x\times 6-6(7-5x) \\
63x-21-28x &= 12x-42+30x \\
35x-42x &= 21-42 \\
-7x &= -21 \\
x &= 21\div 7=3
\end{aligned}$$

(h)

$$\begin{aligned}
x-\left(\frac{2x+8}{3}\right)-\frac{x}{4}+\left(\frac{2-x}{24}\right)+3 &= 0 \\
\frac{24x-8(2x+8)-6x+(2-x)+72}{24} &= 0 \\
24x-16x-64-6x+2-x+72 &= 0 \\
x-64+74 &= 0 \\
x+10 &= 0 \\
x &= -10
\end{aligned}$$

4. (a)

$$\begin{aligned}
0.25p-0.05 &= 0.2p+0.15 \\
0.25p-0.2\,p &= 0.05+0.15
\end{aligned}$$

$$\begin{aligned} 0.05\,p &= 0.20 \\ p &= \frac{0.20}{0.05} = 4 \end{aligned}$$

Check : Substituting $p = 4$ in the given equation, we get

$\text{LHS} = 0.25\,p - 0.05 = 0.25 \times 4 - 0.05 = 1.00 - 0.05 = 0.95$

$\text{RHS} = 0.2\,p + 0.15 = 0.2 \times 4 + 0.15 = 0.8 + 0.15 = 0.95$

$\therefore$ LHS = RHS

Hence, the solution is correct.

(b)

$$\begin{aligned} \frac{(0.25+x)}{3} &= x + \frac{1}{2} \\ \frac{(0.25+x)}{3} &= \frac{2x+1}{2} \\ 2\,(0.25+x) &= 3\,(2x+1)\ \text{[By cross multiplication]} \\ 2 \times 0.25 + 2\,x &= 3 \times 2x + 3 \\ 0.50 + 2x &= 6x + 3 \\ 6x - 2x &= 0.50 - 3 \\ 4x &= -\,2.50 \\ x &= -2.50 \div 4 \\ x &= -\,0.625 = -\frac{625}{1000} = -\frac{5}{8} \end{aligned}$$

(c)

$$\begin{aligned} 0.5\,x + \frac{x}{4} &= 0.25\,x + 0.5 \\ \frac{2.0x + x}{4} &= 0.25\,x + 0.5 \\ \frac{3x}{4} &= 0.25\,x + 0.5 \\ \frac{3x}{4} - 0.25\,x &= 0.5 \\ \frac{3x - 1x}{4} &= 0.5 \\ 2x &= 2.0 \\ x &= 2 \div 2 = 1 \end{aligned}$$

Check : Substituting $x = 1$ in the given equation, we get

$\text{LHS} = 0.5x + \frac{x}{4} = 0.5 \times 1 + \frac{1}{4} = 0.5 + 0.25 = 0.75$

$\text{RHS} = 0.25x + 0.5 = 0.25 \times 1 + 0.5 = 0.25 + 0.5 = 0.75$

$\therefore$ LHS = RHS

Hence, the solution is correct.

(d)

$$\begin{aligned} \frac{12x+5}{2} &= \frac{3x+15}{3} \\ 3\,(12x+5) &= 2\,(3x+15)\ \text{[By cross multiplication]} \\ 3 \times 12x + 3 \times 5 &= 2 \times 3x + 2 \times 15 \\ 36x + 15 &= 6x + 30 \\ 36x - 6x &= 30 - 15 \\ 30x &= 15 \end{aligned}$$

$$\begin{aligned} x &= \frac{15}{30} \\ x &= \frac{1}{2} \end{aligned}$$

Check : Substituting $x = \frac{1}{2}$ in the given equation, we get

$$\text{LHS} = \frac{12x+5}{2} = \frac{12 \times \frac{1}{2} + 5}{2} = \frac{6+5}{2} = \frac{11}{2}$$

$$\text{RHS} = \frac{3x+15}{3} = \frac{3 \times \frac{1}{2} + 15}{3} = \frac{\frac{3}{2} + 15}{3} = \frac{3+30}{6} = \frac{33}{6} = \frac{11}{2}$$

$\therefore$ LHS $=$ RHS

Hence, the solution is correct.

5. (a)

$$\begin{aligned} \frac{5x-3}{2x+6} &= \frac{3}{2} \\ 2(5x-3) &= 3(2x+6)\,\text{[By cross multiplication]} \\ 2 \times 5x - 2 \times 3 &= 3 \times 2x + 3 \times 6 \\ 10x - 6 &= 6x + 18 \\ 10x - 6x &= 6 + 18 \\ 4x &= 24 \\ x &= 24 \div 4 \\ x &= 6 \end{aligned}$$

(b)

$$\begin{aligned} \frac{5x-3}{2x+1} &= \frac{2}{5} \\ 5(5x-3) &= 2(2x+1) \\ 5 \times 5x - 5 \times 3 &= 2 \times 2x + 2 \\ 25x - 15 &= 4x + 2 \\ 25x - 4x &= 15 + 2 \\ 21x &= 17 \\ x &= \frac{17}{21} \end{aligned}$$

(c)

$$\begin{aligned} \frac{3x-4}{2} &= \frac{x+2}{3} \\ 3(3x-4) &= 2(x+2) \\ 3 \times 3x - 3 \times 4 &= 2x + 4 \\ 9x - 12 &= 2x + 4 \\ 9x - 2x &= 12 + 4 \\ 7x &= 16 \\ x &= \frac{16}{7} \end{aligned}$$

(d)

$$\begin{aligned} \frac{9x+1}{3x+5} &= 2 \\ 9x + 1 &= 2(3x+5) \\ 9x + 1 &= 2 \times 3x + 10 \end{aligned}$$

$$\begin{aligned} 9x-6x &= 10-1 \\ 3x &= 9 \\ x &= 9 \div 3 \\ x &= 3 \end{aligned}$$

(e)

$$\begin{aligned} \frac{9y-1}{3y+2} &= \frac{3y-5}{y+6} \\ (9y-1)(y+6) &= (3y-5)(3y+2) \\ & \quad \text{[By cross multiplication]} \\ 9y\times y-y+9y\times 6-6 &= 3y\times 3y+2\times 3y-5\times 3y-10 \\ 9y^2-y+54y-6 &= 9y^2+6y-15y-10 \\ 53y-6 &= -9y-10 \\ 53y+9y &= -10+6 \\ 62y &= -4 \\ y &= -\frac{4}{62} \\ y &= -\frac{2}{31} \end{aligned}$$

(f)

$$\begin{aligned} 1\frac{2}{3}x-\frac{x-1}{4} &= \frac{x-3}{5} \\ \frac{5}{3}x-\frac{x-1}{4} &= \frac{x-3}{5} \\ \frac{5x\times 4-3(x-1)}{12} &= \frac{x-3}{5} \\ \frac{20x-3x+3}{12} &= \frac{x-3}{5} \\ \frac{17x+3}{12} &= \frac{x-3}{5} \\ 5(17x+3) &= 12(x-3) \\ 5\times 17x+15 &= 12x-36 \\ 85x+15 &= 12x-36 \\ 85x-12x &= -36-15 \\ 73x &= -51 \\ x &= \frac{-51}{73} \end{aligned}$$

Exercise 9.2

1. Let the number be x.

$\therefore$ its half $= \frac{1}{2}x$

$\therefore$

$$\begin{aligned} x+\frac{1}{2}x &= 72 \\ \frac{2x+x}{2} &= 72 \\ \frac{3x}{2} &= 72 \end{aligned}$$

$$3x = 72 \times 2$$
$$x = \frac{72 \times 2}{3}$$
$$x = 24 \times 2$$
$$x = 48$$

Hence, the number is 48.

2. Let the number be x.

$$\frac{4}{5}x - \frac{3}{4}x = 3$$
$$\frac{4x \times 4 - 3x \times 5}{20} = 3$$
$$\frac{16x - 15x}{20} = 3$$
$$\frac{x}{20} = 3$$
$$x = 3 \times 20 = 60$$

Hence, the number is 60.

3. Let the three consecutive even number be $x, (x+2)$ and $(x+4)$.

So,
$$x + (x+2) + (x+4) = 36$$
$$x + x + x + 6 = 36$$
$$3x + 6 = 36$$
$$3x = 36 - 6$$
$$x = 30 \div 3$$
$$x = 10$$

The numbers are : $x, x+2, x+4$

$10, 10+2, 10+4$

10, 12, 14

4. Let the two integers be $2x$, $3x$.

Then,
$$2x + 3x = 35$$
$$5x = 35$$
$$x = 35 \div 5$$
$$x = 7$$

So, these integers are : $2x$, $3x$

$2 \times 7, 3 \times 7$

14, 21

5. Let the two numbers be x, y. $(\because x > y)$

Then,
$$x + y = 8 \quad \ldots\text{(i)}$$

And
$$5x = y + 22$$
$$5x - y = 22 \quad \ldots\text{(ii)}$$

Solving the equation (i) and (ii), we get

$x = 5, \ y = 3$

Hence, the numbers is 3 and 5.

6. Let the digit at the unit's place be x

Then, the digit at the ten's place $= y$.

So, $x + y = 10$ …(i)

And $\frac{2}{3}x = y$

$2x - 3y = 0$ …(ii)

Solving the euqation (i) and (ii), we get

$x = 4, \; y = 6$

Hence, the two digits number is 64.

7. Let the digit at the units place be x.

Then, the digit at the ten's place $= y$

$x + y = 7$ …(i)

And if the digits are reversed, then

$$\begin{aligned} 10y + x &= 10x + y + 45 \\ 9y - 9x &= 45 \\ y - x &= 45 \quad \text{…(ii)} \end{aligned}$$

Solving the equation (i) and (ii), we get

$x = 1, y = 6$

Hence, the two digits number is 61.

8. Let the fraction be x/y.

So, $x + 3 = y$

$x - y = -3$ …(i)

And $\frac{x+3}{y+3} = \frac{2}{3}$

$$\begin{aligned} 3x + 9 &= 2y + 6 \quad \text{[By cross multiplication]} \\ 3x - 2y &= -9 + 6 \\ 3x - 2y &= -3 \quad \text{…(ii)} \end{aligned}$$

Solving the equation (i) and (ii), we get

$x = 3, \; y = 6$

Hence, the fraction is $\frac{3}{6}$.

9. Let the age of Mr Kumar be x years.

Then, the age of his son $= y$ years

According to the questions $x + y = 48$ …(i)

After 2 years, $(x + 2) = 3(y + 2)$

$$\begin{aligned} x + 2 &= 3y + 6 \\ x - 3y &= 6 - 2 \\ x - 3y &= 4 \quad \text{…(ii)} \end{aligned}$$

Solving the equation (i) and (ii), we get

$x = 37, \; y = 11$

Hence, their present ages is 37 years and 11 years respectively.

10. Let the present age of father be x years.

Then, my age $= y$ years.

According to the questions

$$\begin{aligned} (x - 4) &= 4(y - 4) \\ x - 4 &= 4y - 16 \\ x - 4y &= 4 - 16 \end{aligned}$$

$$x-4y = -12 \quad \ldots(i)$$
$$x+y = 53 \quad \ldots(ii)$$

Solving the equations (i) and (ii), we get $x = 40, \ y = 13$

Hence, my father's present age is 40 years.

11. Let the length of a rectangle be x cm.

Then, $l = 2b-2$ ...(i)

$\because$ the perimeter of a rectangle $= 2(l+b)$

$\therefore$ the length of a wire $= 20\,\text{cm}$

$$2(l+b) = 20\,\text{cm}$$
$$2(2b-2+b) = 20\,\text{cm}$$
$$3b-2 = 10\,\text{cm}$$
$$b = \frac{12}{3}\,\text{m}$$
$$b = 4\text{ cm}$$

Putting the value of b in equation (i), we get

$$l = 2\times 4-2$$
$$l = 8-2 = 6\,\text{cm}$$

Hence, the length and breadth of the rectangle is 6 cm and 4 cm respectively.

12. Let the fraction be $\frac{x}{y}$.

Then, $4x-1 = y$ ...(i)

$$\frac{x+1}{y+1} = \frac{3}{8}$$
$$8(x+1) = 3(y+1)$$
$$8x+8 = 3y+3$$
$$8x+8 = 3\times(4x-1)+3 \quad \text{[from equation (i)]}$$
$$8x+8 = 12x-3+3$$
$$12x-8x = 8$$
$$4x = 8$$
$$x = 8\div 4 = 2$$

Putting the value of x in equation (i), we get

$$y = 4\times 2-1 = 7$$

Hence, the fraction is $\frac{2}{7}$.

13. We have, $x+y+z = ₹\ 190$ ...(i)

And according to the questions. $y = x+15$ ...(ii)

$z = y+19$ ...(iii)

Putting the value of y, z in equation (i), we get

$$x+(x+15)+(y+19) = ₹\ 190$$
$$2x+15+(x+15+19) = ₹\ 190$$
$$3x+30+19 = ₹\ 190$$
$$3x = ₹\ (190-49)$$
$$3x = ₹\ 141$$
$$x = ₹\ (141\div 3)$$
$$x = ₹\ 47$$

Putting the value of x in equation (ii), we get

$$y = x+15$$
$$y = ₹\ (47+15)$$
$$y = ₹\ 62$$

Putting the value of y in equation (iii), we get

$$z = y+19$$
$$z = ₹\ (62+19)$$
$$z = ₹\ 81$$

Hence, each share, $x = ₹\ 47$, $y = ₹\ 62$ and $z = ₹\ 81$.

14. Let the total distance be x km.

So, $$\frac{2}{5}x+\frac{3}{10}x+\frac{1}{5}x+5 = x$$
$$\frac{2x\times 2+3x\times 1+2\times x+5\times 10}{10} = x$$
$$4x+3x+2x+50 = 10x$$
$$9x+50 = 10x$$
$$10x-9x = 50$$
$$x = 50\,\text{km}$$

Hence, the total distance is 50 km.

15. Let the first part of given amount be ₹ x.

Then, the second part $= ₹\ (200-x)$

So, according to the questions

$$\frac{1}{3}x = \frac{1}{2}(200-x)$$
$$\frac{1}{3}x = ₹\ 100-\frac{x}{2}$$
$$\frac{1}{3}x+\frac{1}{2}x = ₹\ 100$$
$$\frac{2x+3x}{6} = ₹\ 100$$
$$5x = ₹\ 600$$
$$x = ₹\ (600\div 5)$$
$$x = ₹\ 120$$

And second part is ₹ $(200-120) = ₹\ 80$

16. Let the number of 50 paise coins be x.

Then, the number of 25 paise coins $= 2x$.

So, $$2x\times 25+x\times 50 = ₹\ 34$$
$$50x+50x = ₹\ 34$$
$$100x = 34\times 100$$
$$x = \frac{34\times 100}{100} \qquad x = 34\text{ coins}$$

Therefore, the number of 25 paise coins $= 2\times 34 = 68$

The number of 50 paise coins $= x = 34$ coins.

17. Let the cost of a pen and a pencil be ₹ x and ₹ y respectively.

Therefore, $$5x+10y = ₹\ 30$$

$$x+2y = ₹\ 6 \qquad \ldots(i)$$

And $$x = 3+y$$

$$x-y = 3 \qquad \ldots(ii)$$

Solving the equation (i) and (ii), we get

$x = ₹\ 4,\ y = ₹1$

Hence, the cost of a pen and pencil is ₹ 4 and 1 ₹ respectively.

18. Let the number of pens and pencils be x and y respectively.

So, according to the questions

$$7x+3y = ₹\ 564 \qquad \ldots(i)$$

And $$x+y = 108 \qquad \ldots(ii)$$

Solving the equation (i) and (ii), we get

$x = 60,\ y = 48$

Hence, the number of pens and pencils is 60, 48 respectively.

19. Let the cost of a chocolate bar be $2x$.

Then, the cost of a ice-cream $= x$

So,

$$\begin{aligned} 2x\times 2+1\times x &= ₹\ 11.50 \\ 4x+x &= ₹\ 11.50 \\ 5x &= ₹\ 11.50 \\ x &= ₹\ 2.30 \end{aligned}$$

∴ the cost of a ice-cream is ₹ 2.30 and the cost of one chocolate bar is ₹ 2 × 2.30 or ₹ 4.60.

20. Let the number of days be x when he was present.

According to the questions

$$\begin{aligned} 50x-10(30-x) &= 1200 \\ 50x-300+10x &= 1200 \\ 60x &= (1200+300) \\ 60x &= 1500 \\ x &= (1500\div 60) \\ x &= 25 \end{aligned}$$

Therefore, absent days $= (30-25) = 5$ days

21. The quantity of silver to be added $= \dfrac{m(y-x)}{100-y}$ kg

$$\begin{aligned} &= \frac{8\times(40-25)}{100-40}\text{ kg} \\ &= \frac{8\times 15}{60}\text{ kg} = \frac{8}{4}\text{ kg} = 2\text{ kg} \end{aligned}$$

22. The speed of the steamer in still water $= \text{speed}\times(x+y)(x-y)$

$$\begin{aligned} &= 2\times(9+8)(9-8) \\ &= 2\times 17\times 1 = 34\text{ km/h} \end{aligned}$$

MCQs

1. (b) **2.** (c) **3.** (c) **4.** (c) **5.** (b) **6.** (a)
7. (b) **8.** (b) **9.** (c) **10.** (b) **11.** (a) **12.** (a)
13. (b) **14.** (d) **15.** (d)

10

Understanding Shapes

Exercise 10.1

1. Since, sum of the angles of a quadrilateral is 360°.

(a)
$$\begin{aligned} a^\circ + 120^\circ + 70^\circ + 85^\circ &= 360^\circ \\ a^\circ + 275^\circ &= 360^\circ \\ a^\circ &= 360^\circ - 275^\circ \\ a^\circ &= 85^\circ \end{aligned}$$

(b)
$$\begin{aligned} 65^\circ + x^\circ + 140^\circ + 100^\circ &= 360^\circ \\ x^\circ + 305^\circ &= 360^\circ \\ x^\circ &= 360^\circ - 305^\circ \\ x^\circ &= 55^\circ \end{aligned}$$

(c)
$$\begin{aligned} 85^\circ + 95^\circ + 70^\circ + p^\circ &= 360^\circ \\ 250^\circ + p^\circ &= 360^\circ \\ p^\circ &= 360^\circ - 250^\circ \\ p^\circ &= 110^\circ \end{aligned}$$

2. Let the measure of the angles be x, x, x and 90°.

Then,
$$\begin{aligned} x^\circ + x^\circ + x^\circ + 90^\circ &= 360^\circ \\ 3x^\circ + 90^\circ &= 360^\circ \\ 3x^\circ &= 360^\circ - 90^\circ \\ 3x^\circ &= 270^\circ \\ x^\circ &= 270^\circ \div 3 \\ x^\circ &= 90^\circ \end{aligned}$$

Hence, the measure of each angle of the quadrilateral is 90°.

3. Let the measure of each angle be x°.

Then,
$$\begin{aligned} x^\circ + x^\circ + x^\circ + x^\circ &= 360^\circ \\ 4x^\circ &= 360^\circ \\ x^\circ &= 360 \div 4 \\ x^\circ &= 90^\circ \end{aligned}$$

Hence, the measure of each angle is 90°.

4. The sum of the angles of a quadrilateral is 360°.

Let the measure of the angles be x°, $2x^\circ$, $3x^\circ$ and $4x^\circ$. Then,
$$\begin{aligned} x^\circ + 2x^\circ + 3x^\circ + 4x^\circ &= 360^\circ \\ 10x^\circ &= 360^\circ \\ x^\circ &= 36^\circ \end{aligned}$$

Hence, the measures of the four angles are :

$1 \times 36^\circ = 36^\circ$; $2 \times 36^\circ = 72^\circ$; $3 \times 36^\circ = 108^\circ$ and $4 \times 36^\circ = 144^\circ$

5. The sum of the angles of a quadrilateral is 360°.

Let the measure of the angles be $7x^\circ$, $8x^\circ$, $10x^\circ$ and $11x^\circ$.

Then,
$$\begin{aligned} 7x^\circ + 8x^\circ + 10x^\circ + 11x^\circ &= 360^\circ \\ 36x^\circ &= 360^\circ \\ x^\circ &= 360^\circ - 36^\circ \\ x &= 10^\circ \end{aligned}$$

Hence, the measures of the four angles are :

$7 \times 10° = 70°$; $8 \times 10° = 80°$; $10 \times 10° = 100°$ and $11 \times 10° = 110°$

6. Let each angles be x.

$\therefore \quad x° + x° + x° + x° = 360°$

$4x° = 360°$

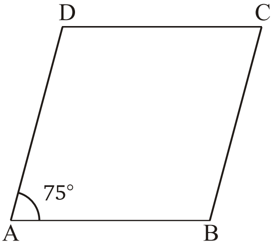
$x° = 360° \div 4$

$x° = 90°$

Hence, the measures of each angle is 90°.

Exercise 10.2

1. In parallelogram $ABCD$



$\angle A = 75°$

Now, $\angle A = \angle C$

[Opposite angles of a parallelogram are equal.]

$\therefore \quad 75° = \angle C$

Also, $\angle A + \angle B = 180°$

[Interior angles on the same side of the transversal.]

$75° + \angle B = 180°$

$\angle B = 180° - 75°$

$\angle B = 105°$

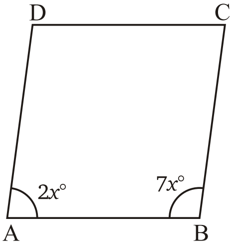
Also, $\angle D = \angle B$

[Opposite angles of a parallelogram are equal.]

$\therefore \quad \angle D = 105°$

Therefore, the measure of three angles of the parallelogram are 105°, 75° and 105°.

2. Let the adjacent angles of the parallelogram $ABCD$ be $2x$ and $7x$.



$2x° + 7x° = 180°$

[Interior angles on the same side of the transversal]

$9x° = 180°$

$x° = 180° \div 9$

$x° = 20°$

Also, $2x° = 2 \times 20° = 40°$

$7x° = 7 \times 20° = 140°$

$\angle C = \angle A$

[Opposite angles of a parallelogram are equal]

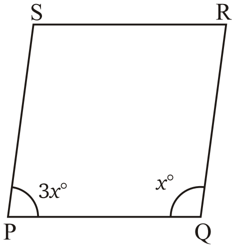
$\therefore \quad \angle C = 40°$

Similarly, $\angle D = \angle B$

$\angle D = 140°$

Hence, the measures of four angles of the parallelogram are 40°, 140°, 40° and 140°.

3. Let the measure of $\angle Q$ be $x°$. Then $\angle P = 3x°$



$\therefore \quad \angle P + \angle Q = 180°$

[Interior angles on the same side of the transversal]

$\therefore \quad 3x° + x° = 180°$

$4x° = 180°$

$x° = 180° \div 4$

$x° = 45°$

Also, $3x^\circ = 3 \times 45^\circ = 135^\circ$

$\angle R = \angle P$

[Opposite angles of a parallelogram are equal.]

$\therefore \quad \angle R = 135^\circ$

Similarly,

$\angle S = \angle Q$

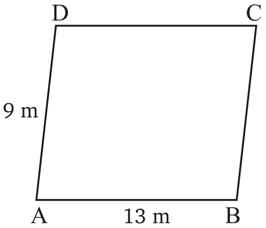
$\angle S = 45^\circ$

Hence, the measure of four angles of the parallelogram are 135°, 45°, 135° and 45°.

4. The perimeter of the parallelogram $ABCD = (AB + BC + CD + DA)$

$= (13 + 9 + 13 + 9)$ m

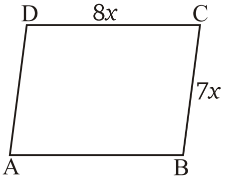
$= 44$ m $\quad [\because AB = DC, BC = AD]$



5. Let $ABCD$ be the parallelogram in which $BC = 7x$ cm and $AB = 8x$ cm.
Since, opposite sides of a parallelogram are equal, therefore,

$AD = BC = 7x$ cm

$AB = DC = 8x$ cm



Perimeter of the parallelogram

$= AB + BC + CD + DA$

$= (7x + 8x + 7x + 8x)$ cm

$= 30x$ cm

Also, perimeter of 11 mm $ABCD = 150$ m [Given]

$30x = 150$

$x = 150 \div 30$

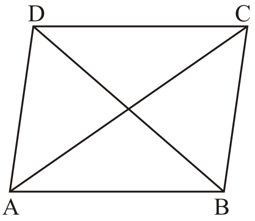
$x = 5$ cm

$\therefore$ length of the sides $= 8 \times 5$ cm $= 40$ cm

and $7x = 7 \times 5$ cm $= 35$ cm

Hence, $AB = 40$ cm, $BC = 35$ cm, $CD = 40$ cm and $DA = 35$ cm are the sides of the parallelogram $ABCD$.

6. No, this is not a true statement.



7. In parallelogram $ABCD$,

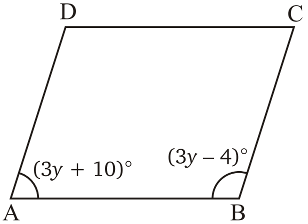
$\angle A + \angle B = 180^\circ$

[Interior angles on the same side of the transversal.]

$\therefore \quad (3y + 10)^\circ + (3y - 4)^\circ = 180^\circ$

$6y^\circ + 6 = 180^\circ$

$(y + 1)^\circ = 180^\circ \div 6$



$$\begin{aligned} (y+1)^\circ &= 30^\circ \\ y^\circ &= 30^\circ - 1^\circ \\ y &= 29^\circ \end{aligned}$$

Also,
$$\begin{aligned} (3y+10)^\circ &= (3\times 29+10)^\circ = 97^\circ \\ (3y-4)^\circ &= (3\times 29-4)^\circ = 83^\circ \\ \angle C &= \angle A \end{aligned}$$

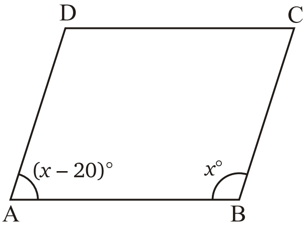
[Opposite angles of a parallelogram are equal.]

$$\therefore \quad \angle C = 97^\circ$$

Similarly,
$$\begin{aligned} \angle D &= \angle B \\ \angle D &= 83^\circ \end{aligned}$$

Hence, the measure of all angles of the parallelogram are 97°, 83°, 97° and 83°.

8. Let the angle of $\angle B$ be x°.



then,
$$\angle A + \angle B = 180^\circ$$

[Interior angles on the same side of the transversal.]

$$\begin{aligned} \therefore \quad 2x^\circ &= 180^\circ - 20^\circ \\ 2x^\circ &= 160^\circ \\ x^\circ &= 80^\circ \\ \angle A &= (x+20)^\circ \\ &= (80^\circ + 20)^\circ = 100^\circ \\ \angle B &= 80^\circ \\ \angle C &= \angle A \end{aligned}$$

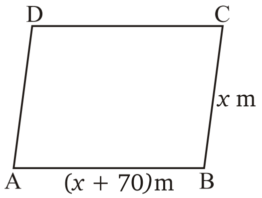
[Opposite angles of a parallelogram are equal.]

$$\therefore \quad \angle C = 100^\circ$$

Similarly,
$$\begin{aligned} \angle D &= \angle B \\ \angle D &= 80^\circ \end{aligned}$$

Hence, the measures of four angles of the parallelogram are 100°, 80°, 100° and 80°.

9.
$$\begin{aligned} \text{Let the one side } &= (x+70)\,\text{m} \\ \text{other side } &= x\,\text{m} \end{aligned}$$



Perimeter of the parallelogram
$$\begin{aligned} &= AB + BC + CD + DA \\ &= (x+70) + x + (x+70) + x \\ &= 4x + 140 \end{aligned}$$

Also, perimeter of || mm $ABCD$
$$\begin{aligned} &= 210\,\text{m [Given]} \\ 4x + 140 &= 210\,\text{m} \\ 4x &= 70\,\text{m} \\ x &= 17.5\text{ m} \\ \therefore \quad \text{length of the side } AB &= (x+70)\,\text{m} \\ &= (17.5 + 70)\text{ m} \\ &= 87.5\text{ m} \end{aligned}$$

And
$$\begin{aligned} BC &= x\,\text{m} \\ &= 17.5\text{ m} \end{aligned}$$

Hence, the length of the sides are 87.5 m, 17.5 m, 87.5 m and 17.5 m.

10. $\angle P = \angle R$ [Opposite angles of a parallelogram are equal.]

$\angle P = 50°$

$\angle Q + \angle P = 180°$

[Interior angles on the same side of the transversal.]

$\therefore \quad \angle Q + 50° = 180°$

$\angle Q = 180° - 50°$

$\angle Q = 130°$

$\angle L + \angle P = 180°$

[$PN \parallel LM$ and interior angles on the same side of the transversal.]

$\angle L + 50° = 180°$

$\angle L = 180° - 50°$

$\angle L = 130°$

Also, $\angle M = \angle P$ [Opposite angles of a parallelogram are equal.]

$\therefore \quad \angle M = 50°$

Hence, the measure of $\angle Q, \angle P, \angle L$ and $\angle M$ are 130°, 50°, 130° and 50° respectively.

11. (a) $\angle A = \angle C$ [Opposite angles of a parallelogram are equal.]

(b) $\angle FAB = \frac{1}{2}\angle A$ [AF bisect $\angle A$]

(c) $\angle DCE = \angle C$ [No, the statement is not true.]

(d) $\angle FAB = \angle DCE$ [Bisect each other]

(e) $\angle DCE = \angle CEB$ [Alternate angles are equal.]

(f) $\angle CEB = \angle FAB$ [$AF \parallel EC$ and corresponding angles]

(g) $CE \parallel AF$ [$\angle FAB = \angle CEB$]

(h) $AE \parallel FC$ [$DC \parallel AB$]

Exercise 10.3

1. Let breadth of the rectangle be b m.

$\because$ the perimeter of the rectangle $= 2(l+b)$

$2(l+b) = 192 \text{ cm}$

$l + b = 96 \text{ cm}$

$62 + b = 96 \text{ cm}$

$b = (96-62) \text{ cm}$

$b = 34 \text{ cm}$

Hence, the breadth of the rectangle is 34 cm.

2. The side of a square $= 45$ cm (Given)

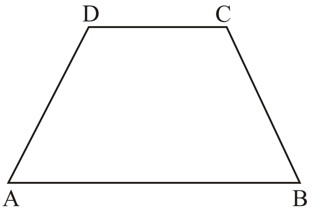
$\because$ the perimeter of the square $= 4$ side

$= 4 \times 45 \text{ cm}$

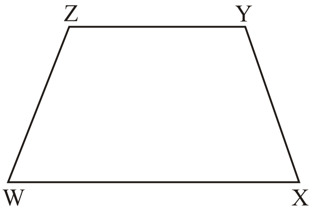
$= 180 \text{ cm}$

3. Since, $AB \parallel DC$ and $AD = BC$ (Given)

So, the measure of other non-parallel side $= 7$ cm.



4. $\because$ $\angle W$ and $\angle Z$ are corresponding angles.



$$\begin{aligned} \angle W+\angle Z &= 180^\circ \\ 3x^\circ+2x^\circ &= 180^\circ \\ 5x^\circ &= 180^\circ \\ x^\circ &= 36^\circ \\ \therefore \quad \angle W &= 3x \\ &= 3\times 36 \\ &= 180^\circ \\ \angle Z &= 2x^\circ = 2\times 36^\circ = 72^\circ \end{aligned}$$

Similarly,

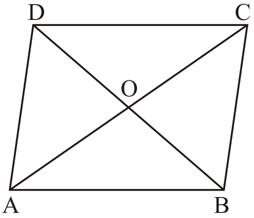
$$\begin{aligned} \angle X+\angle Y &= 180^\circ \\ 4x^\circ+5x^\circ &= 180^\circ \\ 9x^\circ &= 180^\circ \\ x^\circ &= 20^\circ \\ \therefore \quad \angle X &= 4x^\circ = 4\times 20^\circ = 80^\circ \\ \angle Y &= 5x^\circ = 5\times 20^\circ = 100^\circ \end{aligned}$$

Hence, the angles of the trapezium are 108°, 72°, 80° and 100°.

5.

$$\begin{aligned} \text{The perimeter of a square} &= 4 \text{ side} \\ 4 \text{ side} &= 32.4 \text{ m} \\ \text{side} &= 8.1 \text{ m} \end{aligned}$$

6. Consider the given figure, in which *ABCD* is a quarilateral and diagonals *AC* and *BD* bisect each other at *O*, such that $\angle AOB = 90^\circ$.



$$\begin{aligned} AC &= 30\,\text{cm} \\ \Rightarrow \quad AO &= 15\,\text{cm} \\ BD &= 16\,\text{cm} \\ \Rightarrow \quad BO &= 8\,\text{cm} \end{aligned}$$

In right ΔAOB, using pythagoras theorem,

$$\begin{aligned} AB^2 &= AO^2+BO^2 \\ AB^2 &= 15^2+8^2 \\ AB^2 &= 225+64 \\ AB^2 &= 289 \\ AB &= \sqrt{289}\text{ cm} \\ AB &= 17\,\text{cm} \end{aligned}$$

7. Let the length and breadth of a rectangle be $9x$ and $4x$ respectively.

So,

$$\begin{aligned} \text{perimeter of rectangle} &= 130\,\text{cm} \\ 2(9x+4x) &= 130\,\text{cm} \\ 2(13x) &= 130\,\text{cm} \\ 13x &= 65\,\text{cm} \\ x &= (65\div 13)\,\text{cm} \\ x &= 5\,\text{cm} \end{aligned}$$

So,

$$\begin{aligned} \text{the length of a rectangle} &= 9x \\ &= 9\times 5\,\text{cm} \\ &= 45\,\text{cm} \\ \text{the breadth of a rectangle} &= 4x = 4\times 5\,\text{cm} \\ &= 20\,\text{cm} \end{aligned}$$

8. (a) $PQ = RS$ [Opposite sides of a rectangle are equal.]

(b) $\angle QBA = \angle SAR$ [Each angles are 90°.]

(c) $\angle QPB = \angle SRP$ [alternate angle]

(d) $SA = BQ$

$PQ \parallel SR$ and $PQ = SR$

$\angle SAR = \angle QBP = 90°$

So, $\Delta QBP \cong \Delta SAR$ [By SAS congruence condition]

(e) $BQ = SA$ [$PQ = SR$, $PB = AR$]

corresponding parts of congruent triangle.

MCQs

1. (b) **2.** (b) **3.** (a) **4.** (b) **5.** (b)

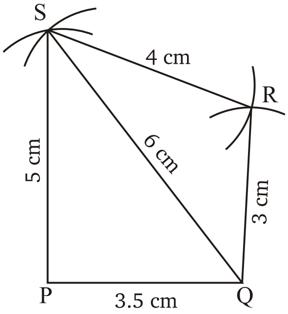
11 Construction of Quadrilaterals

Exercise 11.1

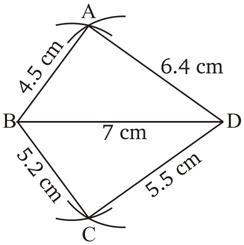
1. Steps of construction :

1. Draw $QS = 6$ cm.
2. With P as centre and a radius equal 5 cm, draw an arc above PS.
3. With Q as centre and a radius equal to 6 cm, draw another arc cutting the first arc at S.
4. With s as centre and radius equal to 4 cm, draw an arc.
5. With Q as centre and radius equal to 3 cm, draw another arc cutting the first arc at R.
6. Join PS, DR, SR.

Then, $PQRS$ is the required quadrilateral.



2. Steps of construction :



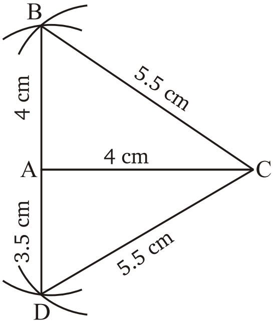
1. Draw $BD = 7$ cm.
2. With B as centre and a radius equal to 4.5 cm, draw an arc above BD.
3. With D as centre and a radius equal to 6.4 cm, draw another arc cutting the first arc at A.
4. With B as centre and radius equal to 5.2 cm, draw an arc below BD.
5. With D as centre and radius equal to 5.5 cm, draw another arc cutting the first arc at C.
6. Join AB, AD, BC and DC.

Then, $ABCD$ is the required quadrilateral.

3. Steps of construction :

1. Draw $AC = 4$ cm.
2. With A as centre and a radius equal to 4 cm, draw an arc above AC.
3. With C as centre and a radius equal to 5.5 cm, draw another arc cutting the first arc at B.
4. With A as centre and radius equal to 3.5 cm, draw an arc below AC.
5. With C as centre and radius equal to 5.5 cm, draw another arc cutting the first arc at D.
6. Join AB, CB, AD and CD.

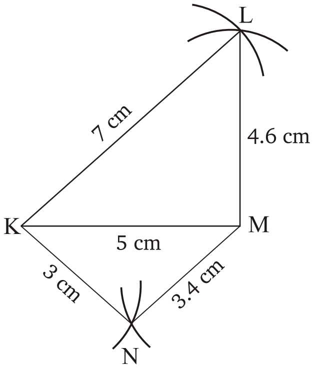
   Then, $ABCD$ is the required quadrilateral.



4. Steps of construction :

1. Draw $KM = 5$ cm.
2. With K as centre and a radius equal to 7 cm, draw an arc above KM.
3. With M as centre and a radius equal to 4.6 cm, draw another arc cutting the first arc at L.
4. With K as centre and a radius equal to 3 cm, draw an arc below KM.
5. With M as centre and a radius equal to 3.4 cm, draw another arc cutting the first arc at N.
6. Join KL, ML, KN and MN.

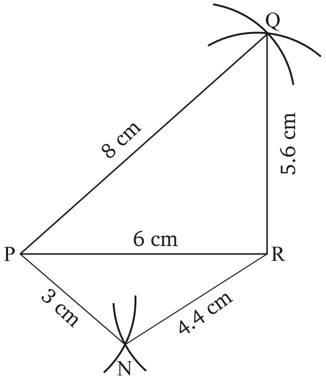
   Then, $KLMN$ is the required quadrilateral.



5. Steps of construction :

1. Draw $PR = 6$ cm.
2. With P as centre and a radius equal to 8 cm, draw an arc above PR.
3. With R as centre and a radius equal to 5.6 cm, draw an another cutting the first arc at Q.
4. With P as centre and a radius equal to 3 cm, draw an arc below PR.
5. With R as centre and a radius equal to 4.4 cm, draw another arc cutting the first arc at S.
6. Join PQ, RQ, PS and RS.

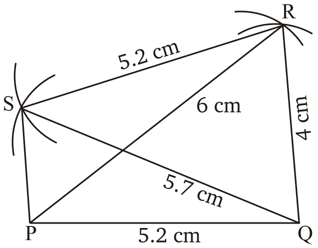
   Then, $PQRS$ is the required quadrilateral.



Exercise 11.2

1. Steps of construction :

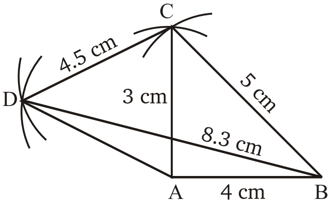
1. Draw $PQ = 5.2$ cm.
2. With P as centre and a radius equal to 6 cm, draw an arc.
3. With Q as centre and a radius equal to 4 cm, draw another arc cutting first arc at R.
4. Join PR and QR.



5. With R as centre and a radius equal to 5.2 cm, draw an arc.
6. With Q as centre and a radius equal to 5.7 cm, draw another arc cutting the first arc at S.
7. Join PS, RS and QS.

   Then, $PQRS$ is the required quadrilateral.

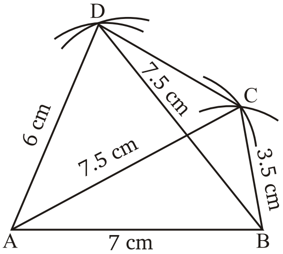
2. Steps of construction :



1. Draw $AB = 4$ cm.
2. With A as centre and a radius equal to 3 cm, draw an arc.
3. With B as centre and radius equal to 5 cm, draw another arc cutting first arc at C.
4. Join AC and BC.
5. With C as centre and radius equal to 4.5 cm, draw an arc.
6. With B as centre and a radius equal to 8.3 cm, draw another arc cutting the first arc at D.
7. Join AD, CD and BD.

   Then $ABCD$ is the required quadrilateral.

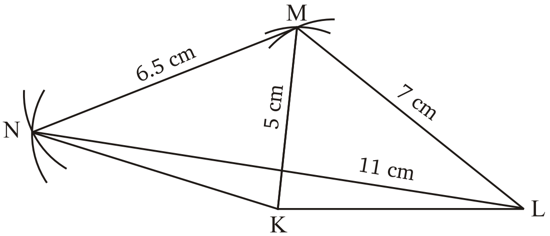
3. Steps of construction :



1. Draw $AB = 7$ cm.
2. With A as centre and a radius equal to 7.5 cm, draw an arc.
3. With B as centre and radius equal to 3.5 cm, draw another arc cutting first arc at C.
4. Join AC and BC.
5. With B as centre and a radius equal to 7.5 cm, draw an arc.
6. With A as centre and a radius equal to 6 cm, draw another arc cutting first arc at D.
7. Join AD, DC and BD.

   Then, $ABCD$ is the required quadrilateral.

4. Steps of constructions :

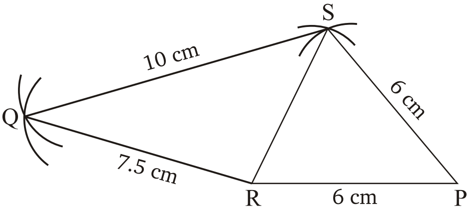


1. Draw $KL = 6$ cm
2. With L as centre and a radius equal to 7 cm, draw an arc.
3. With K as centre and a radius equal to 5 cm, draw another arc cutting first arc at M.
4. Join KM and LM.
5. With L as centre and a radius equal to 11 cm, draw an arc.
6. With M as centre and a radius equal to 6.5 cm, draw another arc cutting first arc at N.
7. Join MN, LN and KN.

   Then, $KLMN$ is the required quadrilateral.

5. Steps of construction :

1. Draw $RP = 6$ cm.



2. With P as centre and a radius equal to 6 cm, draw an arc.
3. With R as centre and a radius equal to 5 cm, draw another arc cutting first arc at S.
4. Join RS and PS.
5. With R as centre and a radius equal to 7.5 cm, draw an arc.
6. With S as centre and a radius equal to 10 cm, draw another arc cutting first arc at Q.
7. Join QR, QS,

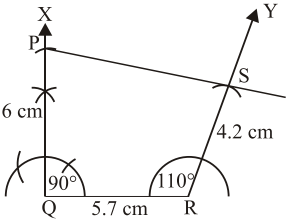
   Then, $PQRS$ is the required quadrilateral.

Exercise 11.3

1. Steps of construction :

1. Draw $QR = 5.7$ cm
2. With Q as centre, draw $\angle XQR = 90°$.
3. From ray QX cut off $PQ = 6$ cm.
4. At R, $\angle QRY = 110°$.
5. From ray RY cut off $RS = 4.2$ cm.
6. Join PS.

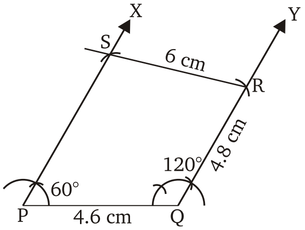
   Thus, $PQRS$ is the required quadrilateral.



2. Steps of construction ;

1. Draw $PQ = 4.6$ cm.
2. With P as centre, draw $\angle XPQ = 60°$.
3. With Q as centre, draw $\angle PQY = 120°$.
4. From ray QY cut off $QR = 4.8$ cm
5. From R cut off $RS = 6$ cm.
6. Join RS.

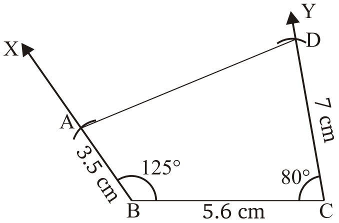
   Thus, $PQRS$ is the required quadrilateral.



3. Steps of construction :

1. Draw $BC = 5.6$ cm.
2. With B as centre, draw $\angle XBC = 125°$.
3. From ray BX cut off $AB = 3.5$ cm.
4. At C, $\angle YCB = 80°$
5. From ray CY cut off $CD = 7$ cm.
6. Join AD.

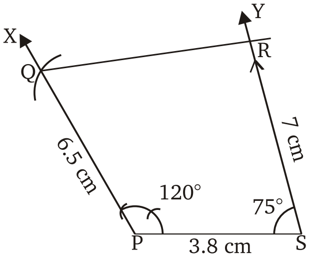
   Thus, $ABCD$ is the required quadrilateral.



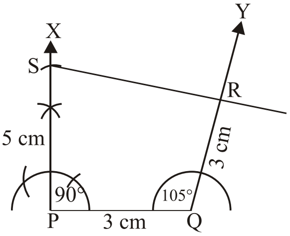
4. Steps of construction :

1. Draw $PS =$ 3.8 cm.
2. With P as centre, draw $\angle XPS = 120°$.
3. From ray XP cut off $PQ = 6.5$ cm.
4. At S, $\angle YSP = 75°$.
5. From ray SY cut off $RS = 7$ cm.
6. Join QR.

   Thus, $PQRS$ is the required quadrilateral.



5. Steps of construction :

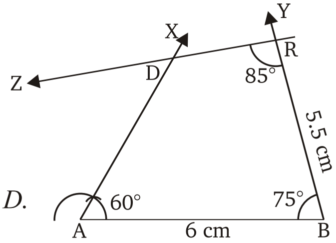


1. Draw $PQ = 3\,\text{cm}$.
2. With P as centre, draw $\angle XPQ = 90$.
3. From ray PX cut off $PS = 5\,\text{cm}$.
4. At $Q, \angle PQY = 105°$.
5. From ray QY cut off $QR = 3\,\text{cm}$.
6. Join RS.

Thus, $PQRS$ is the required quadrilateral.

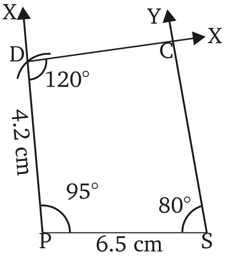
Exercise 11.4

1. Steps of construction :



1. Draw $AB = 6\,\text{cm}$.
2. At A, draw $\angle XAB = 60°$.
3. At B, draw $\angle ABY = 75°$.
4. With B as centre and radius 5.5 cm, draw an arc cutting YB at C.
5. At C, draw $\angle ZCB = 85°$. ZC intersect AY at D.
6. Then, $ABCD$ is the required quadrilateral.

2. Since, sum of angles of a quadrilateral is 360°.



$$\therefore \quad \angle C = 360° - (A + B + D)$$
$$\angle C = 360° - (95° + 80° + 120°)$$
$$\angle C = 65°$$

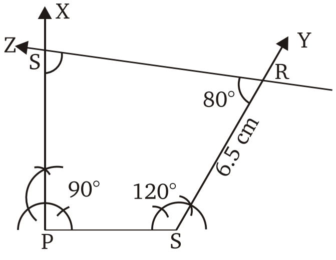
Steps of construction :

1. Draw $AB = 6.5$ cm.
2. With A as centre, draw $\angle XAB = 95°$.
3. At B, draw $\angle YBA = 80°$.
4. With A as centre and radius 42 cm, draw an arc cutting AX at D.
5. At D, draw $\angle ZDA = 120°$, ZD intersects BY at C.
6. Then, $ABCD$ is the required quadrilateral.

3. The given, $\angle P = 90°$, $\angle R = 80°$ and $\angle S = 70°$

$$\therefore \quad \angle Q = 360° - (\angle P + \angle R + \angle S)$$
$$\angle Q = 360° - (90° + 80° + 70°)$$
$$\angle Q = 360° - 240°$$
$$= 120°$$

Steps of construction :



1. Draw $PQ = 3.5$ cm.
2. At P, draw $\angle XPQ = 90°$.
3. At Q, draw $\angle PQY = 120°$.
4. With Q as centre and radius 6.5 cm, draw an arc cutting QY at R.
5. At R, draw$\angle ZRQ = 80°$. ZR intersects PX at S.
6. Then, $PQRS$ is the required quadrilateral.

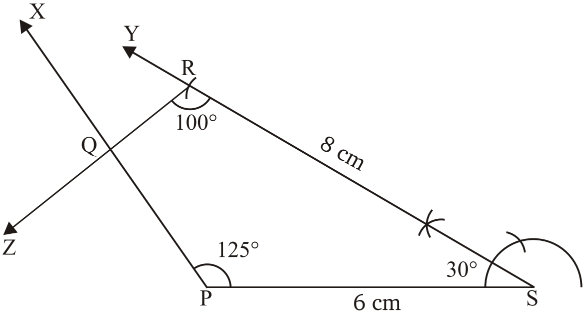
4. The given, $\angle P = 125°, \angle Q = 105°, \angle R = 100°.$

$\therefore \quad \angle S = 360° - (\angle P + \angle Q + \angle R)$

$= 360° - (125° + 105° + 100°)$

$\angle S = 360° - (330°) = 30°$

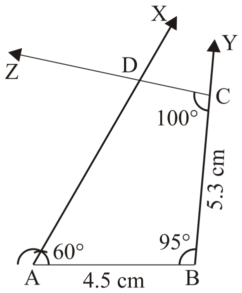
Steps of construction :



1. Draw $PS = 6\,\text{cm}.$
2. At P, draw $\angle XPS = 125°.$
3. At S, draw $\angle PSY = 30°.$
4. With S as centre and radius 8 cm, draw an arc cutting SY at R.
5. At R, draw $\angle ZRS = 100°$, ZR intersects PX at Q.
6. Then, $PQRS$ is the required quadrilateral.

5. Steps of construction :

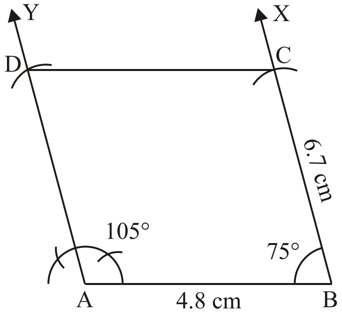
1. Draw $AB = 4.8$ cm.
2. At A, draw $\angle XAB = 60°.$
3. At B, draw $\angle ABY = 95°.$
4. With B as centre and radius 5.3 cm, draw an arc cutting B at C.
5. At C, draw $\angle ZCB = 100°$. ZR intersects AX at D.
6. Then, $ABCD$ is the required quadrilateral.



Exercise 11.5

1. Steps of construction :

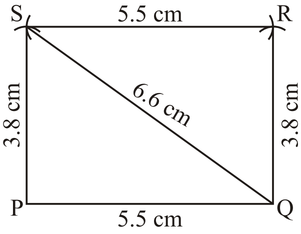
1. Draw$AB = $ 4.8 cm.
2. At B, draw $\angle XBA = 75°.$
3. With B as centre and radius equal to 6.7 cm, draw an arc cutting BX at C.
4. At A, draw $\angle YAB = 105°.$
   $[\because \angle A + \angle B = 180°]$
5. With A as centre and radius equal to 6.7 cm, draw an arc cutting AY at D.
6. Join CD.
   Then, $ABCD$ is the required parallelogram.



2. Steps of construction :

1. Draw $PQ = 5.5$ cm.
2. With Q as centre and radius equal to 6.6 cm, draw an arc.
3. With P as centre and radius equal to 3.8 cm, draw another arc cutting first arc at S. Join QS and PS.
4. With Q as centre and radius equal to 3.8 cm, draw an arc.
5. With S as centre and radius equal to 5.5 cm, draw another arc cutting first arc at R.
6. Join SR and QR.

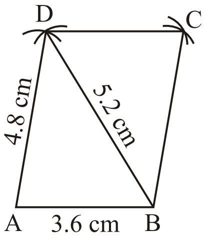
   Then, $PQRS$ is the required parallelogram.



3. Steps of construction :

1. Draw $AB = 3.6$ cm.
2. With A as centre and a radius equal to 4.8 cm, draw an arc.
3. With B as centre and a radius equal to 5.2 cm, draw another arc cutting first arc at D. Join AD and BD.
4. With D as centre and a radius equal to 3.6 cm, draw an arc.
5. With B as centre and a radius equal to 4.8 cm, draw another arc cutting first arc at C. Join DC and BC.

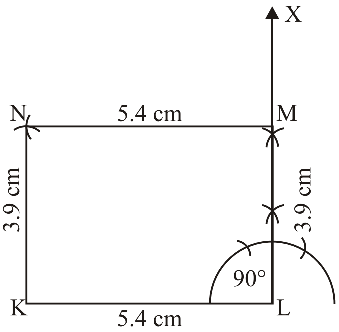
   Then, $ABCD$ is the required parallelogram.



4. Steps of construction :

1. Draw $KL = 5.4$ cm.
2. At L, draw $\angle XLK = 90°$.
3. With L as centre and a radius equal to 3.9 cm, draw an arc cutting on XL at M.
4. With M as centre and a radius equal to 5.4 cm, draw an arc.
5. With K as centre and a radius equal to 3.9 cm, draw another arc cutting first arc at N.
6. Join KN, MN and LM.

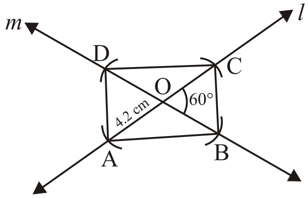
   Thus, $KLMN$ is the required rectangle.



5. Steps of construction :

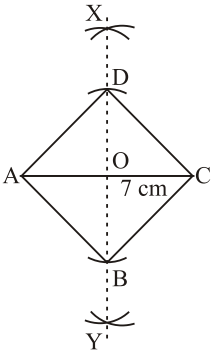
1. Draw a line l.
2. Take a point O any where on line l.
3. With O as centre and draw another line m, making 60° with line l.
4. With O as centre and a radius equal to $\left(\frac{4.2}{2} = 2.1\,\text{cm}\right)$, draw arcs both sices of line l.
5. With O as centre and a radius equal to $\left(\frac{4.2}{2} = 2.1\,\text{cm}\right)$, draw arcs both sides of line m.
6. Join AB, BC, CD and AD.

   Thus, $ABCD$ is the required rectangle.



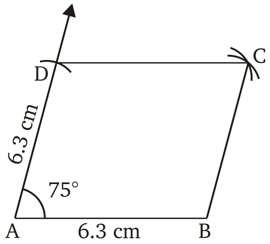
6. Steps of construction :

1. Draw $AC = 7$ cm.
2. Draw XY the perpendicular bisector of AC. XY intersects AC at point O.
3. With O as centre and radius equal to 3.5 cm, $\left(\frac{7}{2} = 3.5 \text{ cm}\right)$, draw an arc cutting OX at D.
4. With O as centre and radius equal to 3.5 cm, draw another arc cutting OY at B.
5. Join AB, BC, CD and AD.
   Then, $ABCD$ is the required square.

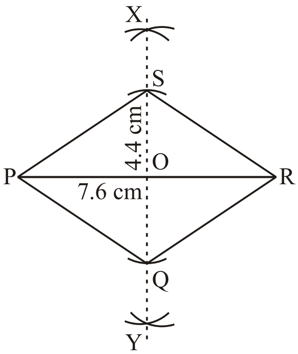


7. Steps of construction :

1. Draw $AB = 6.3$ cm.
2. At A, draw $\angle XAB = 75°$.
3. With A as centre and radius equal to 6.3 cm, draw an arc on AX at D.
4. With D as centre and radius equal to 6.3 cm, draw an arc.
5. With B as centre and radius equal to 6.3 cm, draw another arc cutting first arc at C.
6. Join AB, BC, CD and DA.
   Thus, $ABCD$ is the required rhombus.



8. Steps of construction :



1. Draw $PR = 7.6$ cm.
2. Draw XY the perpendicular bisector of PR. XY intersects PR at point O.
3. With O as centre and radius equal to 2.2 cm, $\left(\frac{4.4}{2} = 2.2 \text{ cm}\right)$, draw an arc cutting OX at S.
4. With O as centre and radius equal to 2.2 cm, draw another arc cutting OY at Q.
5. Join PQ, QR, RS and SP.
   Then, $PQRS$ is the required rhombus.

12

Representing 3D in 2D

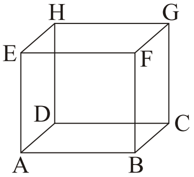
Exercise 12.1

1. (a) Cuboid (b) Cone (c) Cube

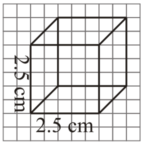
(d) Cylinder (e) Prism (f) Cube

(g) Sphere (h) Cylinder (i) Cylinder

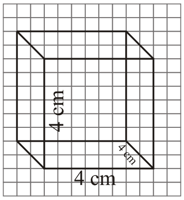
2. $AB, BC, CD, DA, EF, FG, GH, HE, AE,$ BF, CG and DH are 12 edges of the cube.



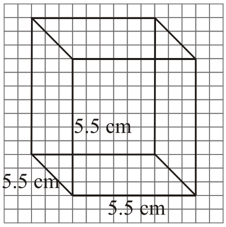
3. (a)



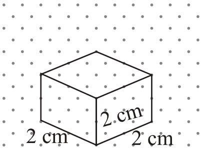
(b)



(c)



4. (a)



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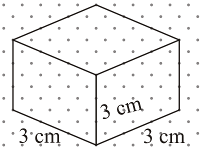
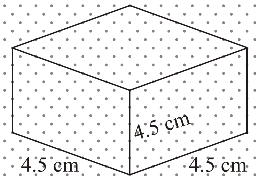
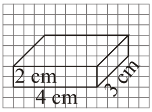


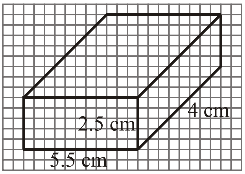
(c)



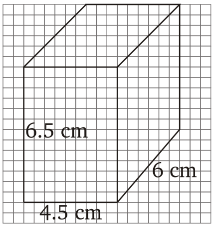
5. (a)



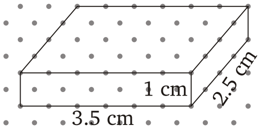
(b)



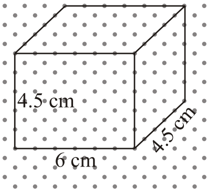
(d)



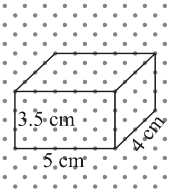
6. (a)



(b)

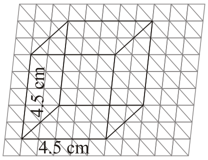


(c)

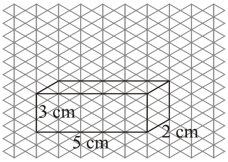


7. Do it yourself

8.



9.



10. Do it yourself

11. (a) cuboid (b) cube (c) rectangular pyramid
(d) triangle prism (e) cylinder (f) triangular octahedron

12. (a) 5 cubes (b) 3 cubes (c) 2 cubes (d) 3 cubes

Exercise 12.2

1.

Solid figures		Vertices
(a)	Triangular prism	6
(b)	Square prism	8
(c)	Hexagonal prism	12
(d)	Pentagonal pyramid	6
(e)	Regular Octahedron	4
(f)	Triangular Pyramid	4

2.

Solid figures		Edges
(a)	Hexagonal prism	18
(b)	Rectangular pyramid	8
(c)	Regular Octahedron	12
(d)	Hexagonal pyramid	12
(e)	Square Prism	12
(f)	Triangular prism	9

3.

Solid figures		Faces
(a)	Square prism	6
(b)	Hexagonal pyramid	7
(c)	Pentagonal prism	7
(d)	Regular octahedron	8
(e)	Sphere	1
(f)	Cylinder	3

4.

Solid	F	E	V	F + V	E + 2
(a)	6	12	8	14	14
(b)	6	10	6	12	12
(c)	7	15	10	17	17
(d)	8	12	6	14	14
(e)	7	12	7	14	14
(f)	5	8	5	10	10
(g)	10	24	16	26	26

5. (a) Faces $= n+1$, Edges $= n \times 2$, and Vertices $= n+1$

(b) Faces $= n+2$, Edges $= n \times 3$ and Vertices $= n+2$

MCQs

1. (b) **2.** (c) **3.** (a) **4.** (b) **5.** (c)

13 Areas

Exercise 13.1

1. The area of a trapezium $= \frac{1}{2}$ (Sum of parallel sides) × Distance between them

$$= \frac{1}{2}(24+16) \times 5 = \frac{1}{2} \times 40 \times 5 = 20 \times 5 = 100\,\text{cm}^2$$

2. Since, the area of a trapezium $= \frac{1}{2}$ (Sum of the parallel sides) × (Distance between them)

$\therefore$ (a) the area of a trapezium $= \frac{1}{2}(a+b) \times h = \frac{1}{2}(40+8) \times 100 = 2400\,\text{cm}^2$

(b) The area of a trapezium $= \frac{1}{2}(a+b) \times h = \frac{1}{2}(14+18) \times 15 = 240\,\text{cm}^2$

(c) The area of a trapezium $= \frac{1}{2}(a+b) \times h = \frac{1}{2}(8.5+3.4) \times 6 = 35.7\,\text{cm}^2$

(d) The area of a trapezium $= \frac{1}{2}(a+b) \times h = \frac{1}{2}(500+50) \times 500 = 137500\,\text{cm}^2$

3. The area of a trapezium $= \frac{1}{2}$ (sum of parallel sides) × (distance between them)

$$= \frac{1}{2}(27+23) \times 19 = 475\,\text{cm}^2$$

4.

$$\begin{aligned}
\text{The area of a trapezium} &= 45\,\text{cm}^2 \\
\text{height} &= 6\,\text{cm} \\
\text{one base} &= 9\,\text{cm} \\
\text{other base} &= ? \\
\text{The area of a trapezium} &= \frac{1}{2}(\text{one base} + \text{other base}) \times \text{height} \\
45 &= \frac{1}{2}(9 + \text{other base}) \times 6 \\
90 \div 6 &= (9 + \text{other base}) \\
15 &= 9 + \text{other base} \\
\text{other base} &= (15-9)\,\text{cm} \\
\text{other base} &= 6\,\text{cm.}
\end{aligned}$$

Hence, the other base is 6 cm.

5.

$$\begin{aligned}
\text{The area of a trapezium} &= 210\,\text{cm}^2 \\
\text{height} &= 14\,\text{cm}
\end{aligned}$$

Let the one parallel side be x cm.

Then, the other parallel side is $2x$ cm.

Therefore, the area of a trapezium $= \frac{1}{2}(x+2x)\times 14$

$$210 = \frac{1}{2}(x+2x)\times 14$$

$$420 = 3x\times 14$$

$$x = 420 \div 42$$

$$x = 10\,\text{cm}$$

One parallel sides $= x = 10\,\text{cm}$

Other parallel side $= 2x = 2\times 10 = 20\,\text{cm}$

Hence, the two parallel sides are 10 cm, 20 cm respectively.

6. Since, ΔBCE is a isosceles triangle.

∴ the area of $\Delta BCE = \frac{1}{2} BE \times$ height

$$210 = \frac{1}{2}\times 6\times \text{height.}$$

$$\text{height} = \frac{210\times 2}{6}\,\text{cm}$$

$$\text{height} = 70\,\text{cm}$$

∴ the area of trapezium $ABCD = \frac{1}{2}(DC + AB)\times$ height

$$= \frac{1}{2}(24+30)\times 70$$

$$= 1890\,\text{cm}^2$$

7. Let the two parallel sides of a trapezium be $5x$ cm and $3x$ cm.

Therefore, the area of a trapezium $= \frac{1}{2}$ (sum of parallel sides) × distance between them

$$960 = \frac{1}{2}(5x+3x)\times 16$$

$$8x = (960 \div 8)\,\text{cm}$$

$$8x = 120\,\text{cm}$$

$$x = 15\,\text{cm}$$

Therefore, the length of one parallel side $= 5x = 5\times 15 = 75\,\text{cm}$

Also, the length of other parallel side $= 3x = 3\times 15 = 45\,\text{cm}$

Hence, the lengths of parallel sides is 75 cm and 45 cm respectively.

8. The area of a trapezium $= 440\,\text{cm}^2$

The parallel sides $= 30\,\text{cm}$, 14 cm

Distance between them $= ?$

∴ The area of a trapezium $= \frac{1}{2}$ (sum of two parallel sides) × distance between them

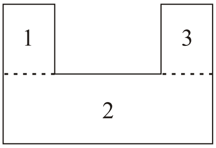
$$440 = \frac{1}{2}(30+14)\times \text{distance between them.}$$

Distance between them $= (880 \div 44)\ \text{cm} = 20\,\text{cm}$

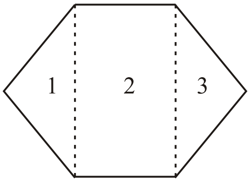
Hence, the distance between two parallel sides is 20 cm.

Exercise 13.2

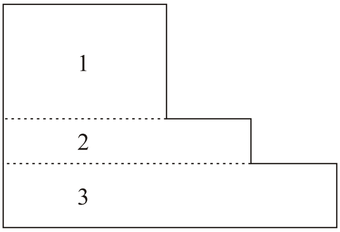
1. (a)



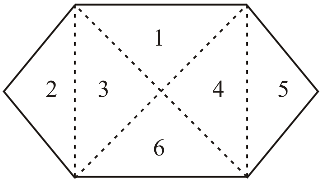
(b)



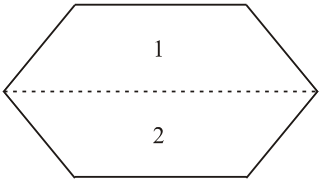
(c)



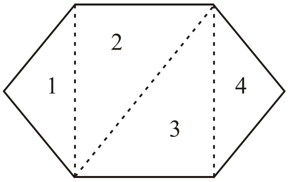
(d)



(e)



(f)



2. (a) The area of given figure

$$= \text{area of } \square ABCD + \text{area of } \Delta ABC$$

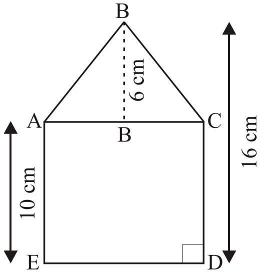
$$= AE \times AC + \frac{1}{2} \times AC \times BF$$

$$= (10 \times 10)\,\text{cm}^2 + \left(\frac{1}{2} \times 10 \times 6\right)\text{cm}^2 \quad (\because AE = AC)$$

$$= (100 + 5 \times 6)\,\text{cm}^2$$

$$= (100 + 30)\,\text{cm}^2$$

$$= 130\,\text{cm}^2$$



(b) The area of given figure :

$$= \text{area of } \square ABCF + \text{area of } \square FCDE$$

$$= (AB \times BC) + \left[\frac{1}{2}(FC + DE) \times \text{height}\right]$$

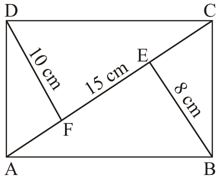
$$= (18 \times 44)\,\text{cm}^2 + \left[\frac{1}{2}(18 + 14) \times 20\right]\text{cm}^2$$

$$= 792\,\text{cm}^2 + \left[\frac{1}{2}(32 \times 20)\right]\text{cm}^2$$

$$= 729\,\text{cm}^2 + 16 \times 20\,\text{cm}^2$$

$$= 792\,\text{cm}^2 + 320\,\text{cm}^2$$

$$= 1112\,\text{cm}^2$$



3. The area of a quadrilateral $\square ABCD$

$= \text{area of } \Delta ADC + \text{area of } \Delta ABC$

$= \frac{1}{2} AC \times DF + \frac{1}{2} AC \times BE$

$= \left(\frac{1}{2} \times 15 \times 10 + \frac{1}{2} \times 15 \times 8 \right) \text{cm}^2 = (75 + 60)\,\text{cm}^2 = 135\,\text{cm}^2$

4. The area of parallelogram $ABCD = DC \times \text{height} = (13 \times 8)\,\text{cm}^2 = 104\,\text{cm}^2$

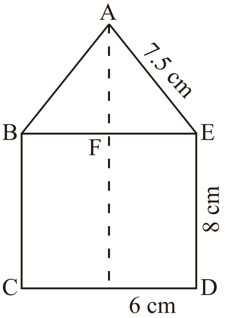
5. The area of a quadrilateral $PQRS$ $= \text{area of } \Delta PRS + \text{area of } PRQ$

$$= \frac{1}{2} PR \times SL + \frac{1}{2}(PR \times QM)$$

$$= \frac{1}{2}(144 \times 28.5 + 144 \times 22)\,\text{cm}^2$$

$$= \frac{1}{2}(4104 + 3168)\,\text{cm}^2$$

$$= \frac{1}{2} \times 4104\,\text{cm}^2 = 3636\,\text{cm}^2$$



6. Since, the given figure are in symmetrical.

So, by Pythagoras theorem,

In ΔAEF,

$$AF^2 = AE^2 - FE^2$$

$$= 7.5^2 - 6^2 \quad (\because FE = MD)$$

$$= (56.25 - 36)\,\text{cm}^2$$

$$= 20.25\,\text{cm}^2$$

$$\therefore \quad AF = 4.5\,\text{cm}$$

Therefore, the area of given figure

$$= \text{area of } BCDE + \text{area of } \Delta ABE$$

$$= BC \times CD + \frac{1}{2} BE \times AF$$

$$= 12 \times 8 + \frac{1}{2} \times 12 \times 4.5 = (96 + 27)\,\text{cm}^2 = 123\,\text{cm}^2$$

MCQs

1. (c) **2.** (c) **3.** (b) **4.** (a) **5.** (d)

14

Volume and Surface Area

Exercise 14.1

1. Since, the volume of cuboid $= l \times b \times h$

(a) The volume of cuboid $= (6.5 \times 3.5 \times 2.0)\,\text{dm}^3 = 45.5\,\text{dm}^3$

(b) The volume of cuboid $= (7.5 \times 4.8 \times 2.6)\,\text{dm}^3 = 93.6\,\text{dm}^3$

(c) The volume of cuboid $= (50 \times 40 \times 16.5)\,\text{m}^3 = 33000\,\text{m}^3$

(d) The volume of cuboid $= (6.2 \times 4.0 \times 3.5)\,\text{m}^3 = 86.80\,\text{m}^3$

(e) The volume of cuboid $= (4.1 \times 2.0 \times 3.7)\ \text{m}^3 = 30.34\ \text{m}^3$

2. The given,

$l = 6.5$ m, $b = 5.6$ m and $h = 35$ cm $= 0.35$ m

$\therefore$ the volume of the wall $= l \times b \times h = (6.5 \times 5.6 \times 0.35)\ \text{m}^3 = 12.74\ \text{m}^3$

3. Since, the capacity of the tank $= l \times b \times h$

(a) The given, $l = 3$ m 75 cm $= 3.75$ cm

$b = 2$ m 80 cm $= 2.80$ m $h = 2$ m 50 cm $= 2.50$ m

$\therefore$ the capacity of the tank $= (3.75 \times 2.80 \times 2.50)\ \text{m}^3 = 26.25\ \text{m}^3$

$(\because 1\ \text{m}^3 = 1000\, l)$

$\therefore$ $26.25 \times 1000 = 26250$ litres

(b) The given,

$l = 2$ m 50 cm $= 2.50$ m $b = 1$ m 60 cm $= 1.60$ m

$h = 1$ m 40 cm $= 1.40$ m

$\therefore$ the capacity of the tank $= (2.50 \times \times 1.60\ \ 1.40)\ \text{m}^3 = 5.60\ \text{m}^3$

or $= (5.60 \times 1000)$ litres $= 5600$ litres

(c) The given,

$l = 3$ m 50 cm $= 3.50$ m $b = 3$ m 50 cm $= 3.50$ m

$h = 1$ m 20 cm $= 1.20$ m

$\therefore$ the capacity of the tank $= (3.50 \times 3.50 \times 1.20)\ \text{m}^3 = 14.70\ \text{m}^3$

or $= 14.70 \times 1000$ litres $= 14700$ litres

4. Since, the volume of cube $= \text{edge}^3$

(a) The given, edge $= 5.2$ cm

$\therefore$ the volume of cube $= \text{edge}^3 = (5.2)^3 = 140.608\ \text{cm}^3$

(b) The given, edge $= 4.5$ cm

$\therefore$ the volume of cube $= (4.5)^3 = 91.125\ \text{cm}^3$

(c) The given, edge $= 1$ dm 6 cm $= (10 + 6)$ cm $= 16$ cm

$\therefore$ the volume of cube $= (16)^3 = 4096\ \text{cm}^3$

5. We know that,

$$\text{The volume of a cube} = (\text{edge})^3$$

$$\text{if the length of edge} = \frac{1}{2}\text{ edge}$$

$$\therefore \quad \text{new volume of a cube} = \left(\frac{1}{2}\text{edge}\right)^3 = \frac{(\text{edge})^3}{8}$$

$$\text{New volume of a cube} = \frac{1}{8}\text{ the volume of a original cube.}$$

Hence,

the volume of a cube will be changed one-eights if the length of its edge is halved.

6. The given, $l = 135$ m, $b = 65$ cm $= 0.65$ m

and $h = 20$ cm $= 0.20$ m

The volume of a path $= l \times b \times h = 135 \times 0.65 \times 0.20\ \text{m}^3 = 17.55\ \text{m}^3$

Hence, $17.55\ \text{m}^3$ of concrete is required to lay a path.

7. The given,

The measures of a wall, $l = 9$ m, $b = 6$ m and $h = 0.25$ m

Therefore, the volume of a wall $= l \times b \times h = 9 \times 6 \times 0.25\ \text{m}^3 = 13.5\ \text{m}^3$

Also, the measures of a brick,

$l = 25\,\text{cm} = 0.25\ \text{m}$ $\qquad b = 11.25\ \text{cm} = 0.1125\ \text{m}$

$h = 8\ \text{cm} = 0.08\ \text{m}$

Therefore, the volume of a brick $= l \times b \times h$

$$= (0.25 \times 0.1125 \times 0.08)\ \text{m}^3 = 0.00225\ \text{m}^3$$

So, the number of brick $= \dfrac{\text{the volume of a wall}}{\text{the volume of a brick}} = \dfrac{13.5}{0.00225} = 6000$

Hence, 6000 bricks will be required to build a wall.

8. The length of a cuboid $= 32\,\text{dm}$

breadth $= 18\,\text{dm}$

height $= 6\,\text{dm}$

Therefore, the volume of a cuboid $= l \times b \times h$

$$= (32 \times 18 \times 6)\,\text{dm}^3 = 3456\,\text{dm}^3$$

Also, the volume of a cube $= (\text{edge})^3 = 18^3 = 5832\,\text{dm}^3$

Difference = the volume of a cube − the volume of a cuboid

$$= (5832 - 3456)\,\text{dm}^3 = 2376\,\text{dm}^3$$

Hence, the difference of a cube and a cubrid is 2376 dm^3.

9. The volume of cubical blocks $= 10 \times 10 \times 10\,\text{cm}^3 = 1000\,\text{cm}^3$

The total volume of wood $= (8 \times 100 \times 75 \times 50)\,\text{cm}^3 = 3000000\,\text{cm}^3$

$\therefore$ number of wooden cubical blocks $= \dfrac{\text{Total volume of wood}}{\text{The volume of cubical block}}$

$$= \frac{3000000}{1000} = 3000$$

Hence, 3000 wooden cubical blocks of size 10 cm can be cut from a log of wood of size 8 m by 75 cm by 50 cm.

10. Let the depth of a tank be *h*m.

$\therefore$ the volume of a tank $= 2000\,\text{m}^3$

$$25 \times 20 \times h = 2000\,\text{m}^3$$

$$h = \frac{2000}{25 \times 20}\ \text{m} = 4\ \text{m}$$

Hence, the depth of a tank is 4 m.

11. Let the height of a box be *h* cm.

$\therefore$ the volume of a box $= l \times b \times h$

$$150 = 6 \times 5 \times h$$

$$h = \frac{150}{6 \times 5}\ \text{cm}$$

$$h = 5\,\text{cm}$$

Hence, the height of a box is 5 cm.

12. The edge of one cube $= \dfrac{1}{2}$ cm.

$\therefore$ the volume of one cube $= \left(\dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2}\right)\text{cm}^3 = \dfrac{1}{8}\ \text{cm}^3$

The volume of a cube with height 5 cm $= (5 \times 5 \times 5)\,\text{cm}^3$

Number of cubes of edge $\frac{1}{2}$ cm

$$= \frac{5 \times 5 \times 5}{\frac{1}{8}} = 125 \times 8 = 1000$$

13. length of two cubes $= (8 + 8)$ cm

$= 16$ cm

breadth $= 8$ cm

height $= 8$ cm

So, the volume of the cuboid $= l \times b \times h = 16 \times 8 \times 8 \text{ cm}^3 = 1024 \text{ cm}^3$

14. The given,

$l = 9$ m, $b = 50$ cm $= 0.50$ m

$h = 20$ cm $= 0.20$ m

The volume of a beam $= 9 \times 0.50 \times 0.20 \text{ m}^3 = 0.9 \text{ m}^3$

$\because$ the weight of 1 m^3 of wood $= 30$ kg

$\therefore$ the weight of 0.9 m^3 of wood $= 30 \times 0.9$ kg $= 27$ kg.

Hence, the weight of the beam is 27 kg.

Exercise 14.2

1. Since, the surface area of the cuboid $= 2((lb + bh + hl)$

(a) The given, $l = 8$ cm, $b = 4$ cm and $h = 2.5$ cm

$\therefore$ the surface area of the cuboid $= 2(lb + bh + hl)$

$= 2(8 \times 4 + 4 \times 2.5 + 2.5 \times 8) \text{ cm}^2$

$= 2(32 + 10 + 20) \text{ cm}^2$

$= 2 \times 64 \text{ cm}^2 = 124 \text{ cm}^2$

(b) The given, $l = 16$ cm, $b = 8$ cm and $h = 5$ cm

$\therefore$ the surface area of the cuboid $= 2(16 \times 8 + 8 \times 5 + 5 \times 16) \text{ cm}^2$

$= 2(128 + 40 + 80) \text{ cm}^2$

$= 2 \times 248 \text{ cm}^2 = 496 \text{ cm}^2$

2. Since, the surface area of a cube $= 6a^2$

(a) The given, $a = 8$ cm

$\therefore$ the surface area of a cube $= 6 \times 8^2 = 6 \times 64 = 384 \text{ cm}^3$

(b) The given, $a = 45$ cm

$\therefore$ the surface area of a cube $= 6 \times 45^2 = 6 \times 2025 = 12150 \text{ cm}^2$

3. The given,

length $= 0.8$ m, breadth $= 65$ cm $= 0.65$ m

and height $= 35$ cm $= 0.35$ m

The area of the cardboard $= 2(lb + bh + hl)$

$= 2(80 \times 65 + 65 \times 35 + 80 \times 35)$

$= 2(5200 + 2275 + 2800) = 20550 \text{ cm}^2$

4. The edge of box $= 15$ cm

The surface area of a cubical wooden box $= 6a^2 = 6 \times 15^2 = 6 \times 225 \text{ cm}^2 = 1350 \text{ cm}^2$

5. The surface area of a cuboid $= 2(lb + bh + hl)$

$$= 2(8 \times 5.8 + 5.8 \times 18 + 18 \times 8)$$

$$= 2(46.40 + 104.4 + 144) = 589.60 \text{ cm}^2$$

6. The given,

$l = 35 \text{ m}$, $b = 30 \text{ m}$ and $h = 8 \text{ m}$

The surface area of a rectangular tank $= 2(lb + bh + hl)$

$$= 2(35 \times 30 + 30 \times 8 + 8 \times 35) \text{ m}^2$$

$$= 2(1050 + 240 + 280) \text{ m}^2$$

$$= 2 \times 1570 \text{ m}^2 = 3140 \text{ m}^2$$

The cost of painting of 1 m^2 = ₹ 2.50

$\therefore$ the cost of painting of 3140 m^2 = ₹ 3140 × 2.50 = ₹ 7850

7. The surface area of wooden box $= 6a^2 = 6 \times 21^2 \text{ cm}^2 = 2646 \text{ cm}^2$

$\because$ cost of painting of 1 $\text{cm}^2 = 50$ paise

$\therefore$ cost of painting of 2646 $\text{cm}^2 = 2646 \times 50 \text{ p} = 132300 \text{ p}$

or = ₹ 1323

8. The surface area of three iron boxes $= [2(1.5 \times 0.75 + 0.75 \times 0.25 + 0.25 \times 1.5)$

$+ 2(1.5 \times 0.60 + 0.60 \times 0.30 + 0.30 \times 15) + 2(2 \times 0.80 + 0.80 \times 0.40 + 0.40 \times 2)] \text{cm}^2$

$= [2(1.125 + 0.1875 + 0.375) + 2(0.9 + 0.18 + 0.45) + 2(1.6 + 0.32 + 0.8)] \text{ cm}^2$

$= [2 \times 1.6875 + 2 \times 1.53 + 2 \times 2.72] \text{ cm}^2$

$= [3.375 + 3.06 + 5.44] \text{ cm}^2 = 11.875 \text{ cm}^2$

$\because$ cost of painting of 1 cm^2 = ₹ 4.0

$\therefore$ cost of painting of 11.875 = ₹ 4.0 × 11.875 = ₹ 47.50

9. Since, two cubes are placed together.

So, total length $= (6 + 6) \text{ cm} = 12 \text{ cm}$

breadth $= 6 \text{ cm}$

height $= 6 \text{ cm}$

$\therefore$ the surface area of the cuboid $= 2(lb + bh + hl)$

$$= 2(12 \times 6 + 6 \times 6 + 6 \times 12) \text{ cm}^2$$

$$= 2(72 + 36 + 72) \text{ cm}^2$$

$$= 2 \times 180 \text{ cm}^2 = 360 \text{ cm}^2$$

10. (a) Let the original side of a cube be x m.

Then, the original surface area of a cube $= 6a^2 = 6x^2 \text{ m}^2$

If the side of a cube is double *i.e.*, $2x$ m.

Then, the new surface area of a cube $= 6(2x)^2 = 6 \times 4x^2 = 4 \times (6x^2) \text{ m}^2$

Hence, the new surface area will be 4 times of the older one.

(b) Let the original side of a cube be y m.

Then, the original surface area of a cube $= 6a^2 = 6y^2 \text{ m}^2$

If the side of a cube is half.

Then, the new surface of a cube $= 6a^2 = 6 \times \left(\frac{1}{2}x\right)^2 = \frac{1}{4}(6x^2)$

Hence, the new surface area will be $\frac{1}{4}$ times of the older one.

11. The ratio of surface areas of two cubes $= 36 : 49$

$$6a_1^2 : 6a_2^2 = 36 : 49$$

$$\left(\frac{a_1}{a_2}\right)^2 = \frac{36}{49}$$

$$a_1 : a_2 = 6 : 7$$

So, the ratio of their volumes $= a_1^3 : a_2^3$

$$= 6^3 : 7^3 = 216 : 343$$

12. The surface area of a cubical box without lid $= 2(bh + hl) + lb$

$$= 2(4 \times 4 + 4 \times 4) + 4 \times 4$$

$$= [2(16 + 16) + 16]\,\text{m}^2$$

$$= (2 \times 32 + 16)\,\text{m}^2$$

$$= (64 + 16)\,\text{m}^2 = 80\,\text{m}^2$$

13. The area of four walls of a room $= 182\,\text{m}^2$

Let the height of wall be h m.

Then, $2h(l + b) = 182$

$$2 \times h(13 + 4.5) = 182$$

$$h(17.5) = 91$$

$$h = 91 \div 17.5$$

$$h = 5.2\text{ m.}$$

14. The perimeter of the floor $= 2(l + b)$

$$2(l + b) = 500$$

$$l + b = 250$$

Area of four walls of the room $= 2h(l + b) = 2 \times 6 \times 250\,\text{m}^2 = 3000\,\text{m}^2$

$\because$ cost of painting of 1 m^2 = ₹ 20

$\therefore$ cost of painting of 3000 m^2 = ₹ 20 × 3000 = ₹ 60000.

Exercise 14.3

1. (a) The given,

$r = 7\,\text{cm}$, $h = 5\,\text{cm}$

Curved surface area of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 5\,\text{cm}^2$$

$$= 44 \times 5\,\text{cm}^2$$

$$= 220\,\text{cm}^2$$

Area of a base $= \pi r^2$

$$= \frac{22}{7} \times 7^2 = 22 \times 7 = 154\,\text{cm}^2$$

Whole surface area $= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 7 \times (7 + 5)\,\text{cm}^2$$

$$= 44 \times 12\,\text{cm}^2 = 528\,\text{cm}^2$$

(b) The given,

$r = 21\,\text{cm},\ h = 15\,\text{cm}$

Curved surface area of the cylinder $= 2\pi rh = 2\times\frac{22}{7}\times 21\times 15 = 1980\,\text{cm}^2$

Area of base $= \pi r^2$

$= \frac{22}{7}\times(21)^2\ \text{cm}^2 = 1386\,\text{cm}^2$

Whole surface area $= 2\pi r(r+h)$

$= 2\times\frac{22}{7}\times 21\times(21+15)\,\text{cm}^2$

$= (44\times 3\times 36)\,\text{cm}^2 = 4752\,\text{cm}^2$

(c) The given, $r = 14$ cm, $h = 10\,\text{cm}$

Curved surface area of the cylinder $= 2\pi rh$

$= 2\times\frac{22}{7}\times 14\times 10\,\text{cm}^2$

$= 44\times 2\times 10\,\text{cm}^2 = 880\,\text{cm}^2$

Area of base $= \pi r^2 = \frac{22}{7}\times(14)^2$

$= 22\times 2\times 14\ \text{cm}^2 = 616\,\text{cm}^2$

Whole surface area $= 2\pi r(r+h)$

$= 2\times\frac{22}{7}\times 14\times(14+10)\,\text{cm}^2$

$= (44\times 2\times 24)\,\text{cm}^2 = 2112\,\text{cm}^2$

(d) The given,

$r = 35$ cm, $h = 8$ cm

Curved surface area of the cylinder $= 2\pi rh$

$= \left(2\times\frac{22}{7}\times 35\times 8\right)\text{cm}^2$

$= (44\times 5\times 8)\,\text{cm}^2 = 1760\,\text{cm}^2$

Area of the base $= \pi r^2$

$= \frac{22}{7}\times(35)^2\ \text{cm}^2$

$= (22\times 5\times 35)\,\text{cm}^2 = 3850\,\text{cm}^2$

Whole surface area $= 2\pi r(r+h)$

$= 2\times\frac{22}{7}\times 35\times(35+8)\,\text{cm}^2$

$= (44\times 5\times 43)\,\text{cm}^2 = 9460\,\text{cm}^2$

Therefore, the table are :

	Radius (cm)	Height (cm)	Curved surface	Area of the base	Whole surface
(a)	7	5	220 cm^2	154 cm^2	528 cm^2
(b)	21	15	1980 cm^2	1386 cm^2	4752 cm^2

(c)	14	10	880 cm^2	616 cm^2	2112 cm^2
(d)	35	8	1760 cm^2	3850 cm^2	4960 cm^2

2. The given,

$$D_1 : D_2 = 2:3$$

Since, the volume of cylinder $= \pi r^2 h$

∴ the volume of first cylinder $= \pi r_1^2 h_1$

Also, the volume of second cylinder $= \pi r_2^2 h_2$

$$\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\frac{h_1}{h_2} = \frac{r_2^2}{r_1^2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{D_2/2}{D_1/2}\right)^2$$

Thus, $h_1 : h_2 = D_2^2 : D_1^2 = 2^2 : 3^2 = 4:9$

Hence, the ratio of their heights is 4 : 9.

3. The curved surface area of a cylindrical pole $= 264 \text{ m}^2$

∴ $2\pi rh = 264 \text{ m}^2$

$\pi rh = 132 \text{m}^2$ …(i)

Also, the volume of a cylindrical pole $= 924 \text{ m}^3$

∴ $\pi r^2 h = 924 \text{ m}^3$

$132\, r = 924 \text{ m}$ [from equation (i)]

$r = 7 \text{m}$

Putting the value of r in equation (i), we get

$$\pi rh = 264$$

$$\frac{22}{7} \times 7 \times h = 264$$

$$h = (264 \div 22) \text{m}$$

$$h = 12 \text{m}$$

4. The given,

$$r_1 : r_2 = 2:3$$

Also, $h_1 : h_2 = 5:4$

Since, the volume of cylinder $(V) = \pi r^2 h$

∴ $V_1 : V_2 = \pi r_1^2 h_1 : \pi r_2^2 h_2$

$$= \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right) = \left(\frac{2}{3}\right)^2 \left(\frac{5}{4}\right)$$

$$= \frac{4}{9} \times \frac{5}{4} = \frac{5}{9}$$

Thus, $V_1 : V_2 = 5 : 9$

5. The given,

$$r_1 : r_2 = 4:5$$

Also, $h_1 : h_2 = 5:6$

∴ $V_1 : V_2 = \pi r_1^2 h_1 : \pi r_2^2 h_2$

$$=\left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right)$$

$$=\left(\frac{4}{5}\right)^2\left(\frac{5}{6}\right)=\frac{16}{25}\times\frac{5}{6}=8:15$$

Thus, $V_1:V_2=8:15$

6. The volume of a cylinder $=6358\,\text{cm}^3$

$\therefore$ $\pi r^2 h=6358\,\text{cm}^3$

$$\frac{22}{7}\times r^2\times 28=6358\,\text{cm}^3$$

$$r^2=\frac{289}{4}\,\text{cm}^2$$

$$r=\sqrt{\frac{289}{4}}\,\text{cm}$$

$$r=\frac{17}{2}\,\text{cm}=8.5\,\text{cm}$$

So, the curved surface area $=2\pi rh$

$$=\left(2\times\frac{22}{7}\times\frac{17}{2}\times 28\right)\text{cm}^2$$

$$=(22\times17\times4)\,\text{cm}^2=1496\,\text{cm}^2$$

7. The given, $r=0.14$ m, $h=1$m

$\because$ The covered area of a roller in 1 revolution $=2\pi rh$

$$=\left(2\times\frac{22}{7}\times0.14\times1\right)\text{cm}^2$$

$$=0.88\,\text{cm}^2$$

$\therefore$ the curved area of a roller in 5 revolution $=5\times0.88\,\text{cm}^2=4.4\,\text{m}^2$

8. $D=28\,\text{cm}$ and $r=R-4$

$2R=28\,\text{cm}$, $r=(14-4)\,\text{cm}=10\,\text{cm}$

$R=14\,\text{cm}$

Also, $h=49\,\text{cm}$

The volume of a hollow cylindrical metal pipe $=\pi(R^2-r^2)h$

$$=\frac{22}{7}(14^2-10^2)\times49\,\text{cm}^3$$

$$=\frac{22}{7}\times(196-100)\times49\,\text{cm}^3$$

$$=\left(\frac{22}{7}\times96\times49\right)\text{cm}^3$$

$$=(22\times96\times7)\,\text{cm}^3$$

$$=14784\,\text{cm}^3$$

$\because$ The weight of 1 cm^3 of a hollow pipe $=8\,\text{g}$

$\therefore$ the weight of 14784 cm^3 of a hollow pipe $=(14784\times8)\,\text{g}$

$$=118272\,\text{g}=118.272\,\text{kg}$$

9. The given,

$d = 8.4$ cm

$\therefore \quad r = 4.2$ cm, $h = 20$ cm

$$\begin{aligned}\text{The volume of a tap } &= \pi r^2 h \\ &= \left(\frac{22}{7} \times (4.2)^2 \times 20\right) \text{cm}^3 \\ &= \left(\frac{22}{7} \times 17.64 \times 20\right) \text{cm}^3 \\ &= \frac{7761.60}{7} \text{cm}^3 \\ &= 1108.80 \text{ cm}^3\end{aligned}$$

The capacity in litres of a cylindrical vessel $= 1108.80 \times 1000$ litre $= 1.1$ litre.

10. The given, $h = 7$ m, $r = 6$ mm $= \frac{6}{1000}$ m

$$\begin{aligned}\therefore \quad \text{the curved surface area of the wire } &= 2\pi rh \\ &= \left(2 \times \frac{22}{7} \times \frac{6}{1000} \times 7\right) \text{m}^2 \\ &= 0.264 \text{ m}^2\end{aligned}$$

11.

$$\begin{aligned}\text{The volume of a cube } &= (11)^3 \text{ cm}^3 \\ &= 1331 \text{cm}^3 \\ \because \quad \text{the volume of a wire } &= \text{the volume of a cube} \\ \pi r^2 h &= 1331 \text{cm}^3 \\ \frac{22}{7} \times (0.5)^2 \times h &= 1331 \text{cm}^3 \\ h &= \frac{1331 \times 7}{22 \times 0.25} \text{cm} \\ h &= 1694 \text{ cm} \\ h &= 16.94 \text{ m}\end{aligned}$$

Hence, 16.94 m of wire will be obtained from the cube.

12. The given,

$d = 1.4$ m and $h = 10$ m

or $r = 0.7$ m

(a)

$$\begin{aligned}\text{Inner curved surface area } &= 2\pi rh \\ &= \left(2 \times \frac{22}{7} \times 0.7 \times 10\right) \text{m}^2 \\ &= (44 \times 1) \text{m}^2 = 44 \text{ m}^2\end{aligned}$$

(b) the cost of plastering the inner curved surface $=$ ₹ $50 \times 44 =$ ₹ 2200.

MCQs

1. (c) **2.** (b) **3.** (a) **4.** (c) **5.** (a)

15

Playing with Numbers

Exercise 15.1

1. Since, $x = 29$ day in Feb

$\therefore$ $29 \div 7 = 1$ remainder

$\therefore$ 1 March of the year will be Tuesday.

So, it is not true.-

2. Let the two numbers be $10a + b$ and $10c + d$ respectively.

Now, $b \times d = 24$ [Given]

Only one pair of digits can give the product 24, *i.e.*, $4 \times 6 = 24$

Here, there are two possibilities :

Either $b = 4$, $d = 6$ or $b = 6$, $d = 4$ …(i)

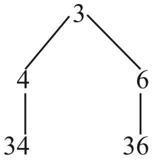
Also, $a \times c = 15$ (Given)

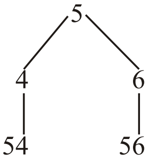
Only one pair of digits can give the product 15 *i.e.*, $3 \times 5 = 15$

Again, there are two possibilities :

Either $a = 3$, $c = 5$ or $a = 5$, $c = 3$ …(ii)

From (i) and (ii), we have four possible numbers :





Now,

$$\begin{array}{r} 34 \\ \times\ 56 \\ \hline 204 \\ 170\times \\ \hline 1904 \\ \hline \end{array}$$

and

$$\begin{array}{r} 36 \\ \times\ 54 \\ \hline 144 \\ 180\times \\ \hline 1944 \\ \hline \end{array}$$

Hence, the required numbers are 56 and 34.

3. Yes;

$$3 = 2^2 - 1$$
$$6 = 2^2 + 1^2 + 1^2$$
$$14 = 3^2 + 2^2 + 1^2$$

4. The given, $x * y = x \times y + 2$

And, $x \sim y = x + y - 1$

So, $\{(3 * 3) * 3\} \sim 3 = \{(3 \times 3 + 2) * 3\} \sim 3$

$= \{11 * 3\} \sim 3$

$= [11 \times 3 + 2\} \sim 3$

$= \{33 + 2\} \sim 3$

$= 35 \sim 3 = 35 + 3 - 1 = 37$

5. Let Sumit's scored be x.

Then, $18x = 36$

$x = 2$

So, Sumit's scored is 2.

Let Vikas's scored be y.

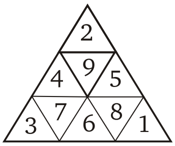
Then, $36 = y + 36$ $\qquad y = 36 - 36 = 0$

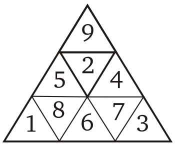
Hence, Raman's scored $= 36$, Sumit's scored $= 2$ and Vikash scored $= 0$

6.

	83	
29	30	64
	10	

7. Two other arrangements of the numbers are :





8. (a) 12, 34 and 56 are divisible by 2. (b) 123 and 456 are divisible by 3.
(c) 1236 is divisible by 4. (d) 12345 is divisible by 5.
(e) 123456 is divisible by 6.

9. (a) In the ones column 'x' can be any numbers from 0 to 9.

Since, $x+3 = 7$,

Therefore, $x = 4$

Similarly, in the tens column 'y' can be any number from 0 to 9.

Since, $y+2 = 8$

$y = 8-2 = 6$

(b) In the ones column 'x' can be any number from 0 to 9.

Since, $9-x = 4$

$x = 9-4 = 5$

Also, in the tens column 'y' can be any number from 0 to 9.

Since, $10-5 = y$

$y = 5$

Also, in the hundreds column $7-z = 2$

$z = 7-2 = 5$

(c) In the ones column 'y' can be any number from 0 to 9.

Since, $3-(2) = y$

$y = 1$

Similarly, in the tens column 'x' can be any number from 0 to 9.

Since, $7-x = 5$

$x = 7-5 = 2$

Similarly, in the hundreds column $= 6-3 = z$

$z = 3$

10. Do it yourself. **11.** Do it yourself.

12. Three different ways :

(i) $98-7-6+5+4+3+2+1=100$ (ii) $98-7+6+5+4-3-2-1=100$

(iii) $98+7-6-5+4+3-2+1=100$

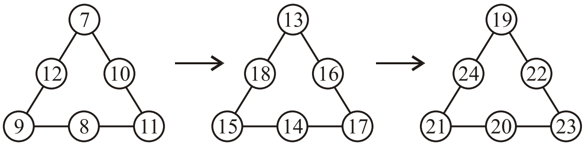
13. Two different integers for x is 2 and -6.

Because
$$\frac{x}{3} = \frac{4}{x+4}$$
$$x(x+4) = 12$$

$$\begin{aligned} x^2+4x-12 &= 0\\ x^2+6x-2x-12 &= 0\\ x(x+6)-2(x+6) &= 0\\ (x+6)(x-2) &= 0 \end{aligned}$$

So, $x = 2, -6$

14. Three possibilities are :



15. (a) Taking the unit digit of both sides.

$$\begin{aligned} x\times 4 &= 8\\ x &= 8\div 4 = 2 \end{aligned}$$

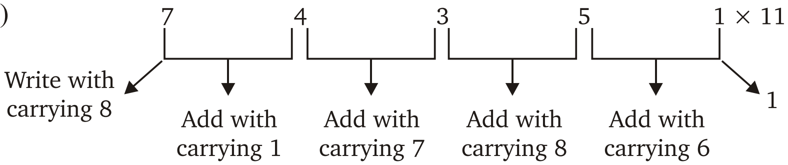
Hence, the value of x is 2.

(b)

$$\begin{aligned} 2\times x^2 \div x &= 8\\ 2\times x^2\times \frac{1}{x} &= 8\\ 2x &= 8\\ x &= 4 \end{aligned}$$

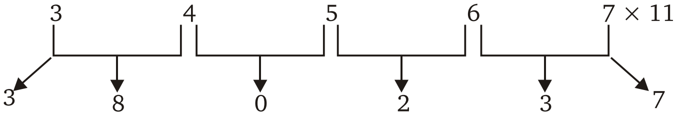
Hence, the value of x is 4.

16. (a)



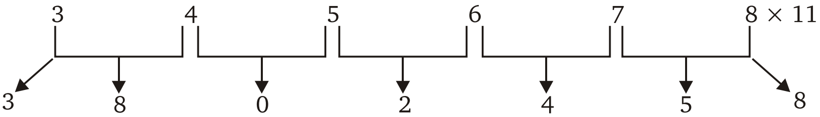
Thus, $74351\times 11 = 817861$

(b)



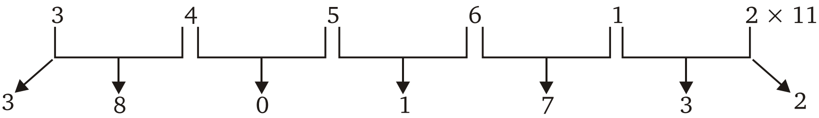
Thus, $34567\times 11 = 380237$

(c)



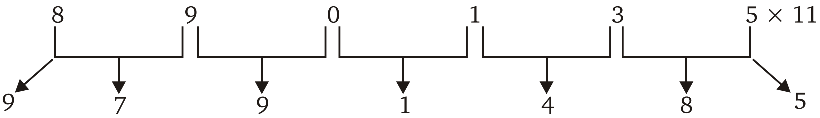
Thus, $345678\times 11 = 3802458$

(d)



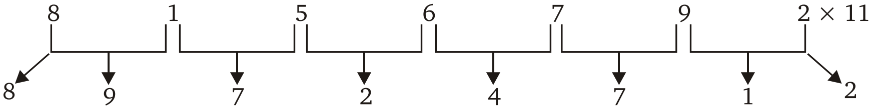
Thus, $345612\times 11 = 3801732$

(e)



Thus, $890135\times 11 = 9791485$

(f)



Thus, 8156792 × 11 = 89724712

17. (a)
$$\begin{array}{r} \text{MARS}\\ \text{BARS}\\ +\text{ARE}\\ \hline \text{BEST}\\ \hline \end{array} \longrightarrow \begin{array}{r} 0314\\ 8314\\ +319\\ \hline 8947\\ \hline \end{array}$$

(b)
$$\begin{array}{r} \text{SANTA}\\ -\text{CLAUS}\\ \hline \text{XMAS}\\ \hline \end{array} \longrightarrow \begin{array}{r} 24794\\ -16452\\ \hline 8342\\ \hline \end{array}$$

(c)
$$\begin{array}{r} \text{JUNI}\\ +\text{JULY}\\ \hline \text{APRII}\\ \hline \end{array} \longrightarrow \begin{array}{r} 8659\\ +8640\\ \hline 17299\\ \hline \end{array}$$

18. (a)

$\xrightarrow{-2}$

$\uparrow +4$

22	20	18	16	14
26	24	22	20	18
30	28	26	24	22
34	32	30	28	26
38	36	34	32	30
42	40	38	36	32

The diagonal increase by 2.

(b)

$\xrightarrow{\times 3}$

$\uparrow \times 2$

2	6	18	54	162
4	12	36	108	324
8	24	72	216	648
16	48	144	432	1296
32	96	288	864	2592
64	192	576	1728	5184

The diagonal multiple by 6.

19. (a)
$$\begin{array}{r} 45\\ \times 45\\ \hline 225\\ 180\times\\ \hline 2025\\ \hline \end{array}$$

Product of tens digits and the next whole number,

i.e., $4 \times 5 = 20$

Product of units digits *i.e.*, $5 \times 5 = 25$

$\therefore$ $45 \times 45 = 2025$

(b)
$$\begin{array}{r} 34\\ \times 36\\ \hline 144\\ 108\times\\ \hline 1224\\ \hline \end{array}$$

Product of tens digits and the next whole number

i.e., $3 \times 4 = 12$

Product of units digit *i.e.*, $6 \times 4 = 24$

$\therefore$ $36 \times 34 = 1224$

(c)
$$\begin{array}{r} 53\\ \times 57\\ \hline 371\\ 265\times\\ \hline 3021\\ \hline \end{array}$$

Product of tens digits and the next whole number,

i.e., $5 \times 6 = 30$

Product of units digits, *i.e.*, $3 \times 7 = 21$

$\therefore$ $53 \times 57 = 3021$

(d)
$$\begin{array}{r} 62\\ \times 68\\ \hline 496\\ 372\times\\ \hline 4216\\ \hline \end{array}$$

Product of tens digits and the next whole number,

i.e., $6 \times 7 = 42$

Product of units digits, *i.e.*, $8 \times 2 = 16$

$\therefore$ $62 \times 68 = 4216$

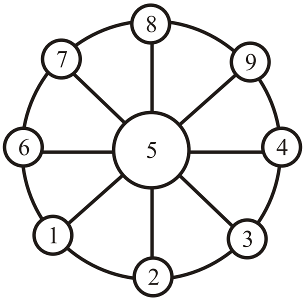
(e)
$$\begin{array}{r} 9\,2 \\ \times 9\,8 \\ \hline 7\,3\,6 \\ 8\,2\,8\times \\ \hline 9\,0\,1\,6 \\ \hline \end{array}$$

Product of tens digits and the next whole number,
i.e., $9\times 10=90$
Product of units digits, *i.e.*, $8\times 2=16$
$\therefore$ $92\times 98=9016$

(f)
$$\begin{array}{r} 8\,2 \\ \times 8\,8 \\ \hline 6\,5\,6 \\ 6\,5\,6\times \\ \hline 7\,2\,1\,6 \\ \hline \end{array}$$

Product of tens digits and the next whole number,
i.e., $8\times 9=72$
Product of units digits, *i.e.*, $2\times 8=16$
$\therefore$ $82\times 88=7216$

20.



21. Do it yourself.
22. Do it yourself.

Exercise 15.2

1. We know that a number is divisible by 2. if its units digit is one of 0, 2, 4, 6, 8. Thus, 62, 98, 3156, 8416 and 5090 are divisible by 2.
2. We know that a number is divisible by 3 if the sum of its digit is divisible by 3.
   Sum of the digits of 625= $6+2+5=13$, which is not divisible by 3.
   Hence, 625 is not divisible by 3.
   Sum of the digits of $852=8+5+2=15$.
   Which is divisible by 3.
   Hence, 852 is divisible by 3.
   Sum of the digits of $7628=7+6+2+8=22$, which is not divisible by 3.
   Hence, 7628 is not divisible by 3.
   Sum of the digits of $7027=7+0+2+7=16$, which is not divisible by 3.
   Hence, 7027 is not divisible by 3.
   Sum of the digits of $92604=9+2+6+0+4=21$, which is divisible by 3.
   Hence, 92604 is divisible by 3.
3. We know that a number is divisible by 5 if its units digit is either 0 or 5.
   Hence, 835, 6025, 3880, 60280.
4. We know that a number is divisible by 9 if the sum of its digits is divisible by 9.
   Sum of the digits of $470=4+7+0=13$, which is not divisible by 9.
   Hence, 470 is not divisible by 9.
   Sum of the digits of $783=7+8+3=18$, which is divisible by 9.
   Hence, 783 is divisible by 9.

Sum of the digits of 1530 $= 1 + 5 + 3 + 0 = 9$, which is divisible by 9.
Hence, 1530 is divisible by 9.
Sum of the digits of 10532 $= 1 + 0 + 5 + 3 + 2 = 11$, which is not divisible by 9.
Hence, 10532 is not divisible by 9.
Sum of the digits of 760298 $= 7 + 6 + 0 + 2 + 9 + 8 = 32$, which is not divisible by 9.

5. We know that a number is divisible by 10 if its units digit is 0.
Hence, 280, 480, 700, 860 and 400 are divisible by 10.

MCQs

1. (c) **2.** (d) **3.** (c) **4.** (b) **5.** (b)

16

Introduction to Graphs

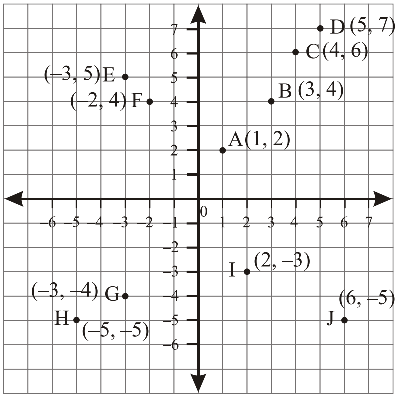
Exercise 16.1

1. (a) 5 (b) −2 (c) 9 (d) 0

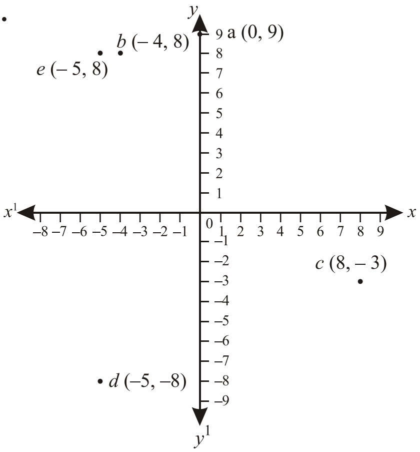
2. (a) 9 (b) 7 (c) 0 (d) −9

3. (a) I quadrant (b) II quadrant (c) IV quadrant
(d) III quadrant (e) II quadrant

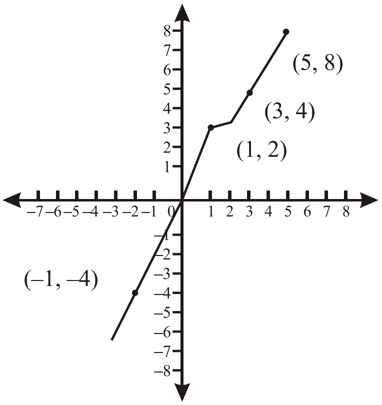
4.



5.



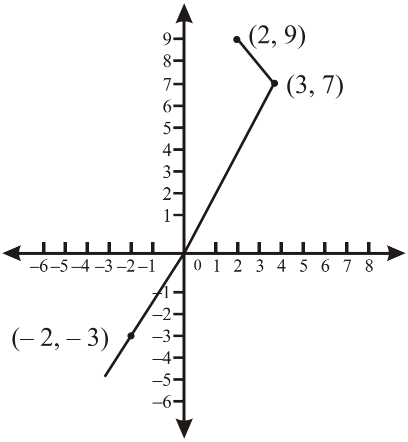
6.



(a) No, (b) $(-1, -4)$

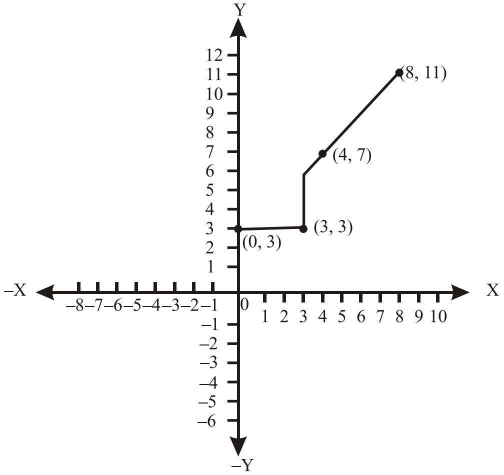
7. (a)

x	$y = 2x + 1$
-2	-3
3	7
2	9



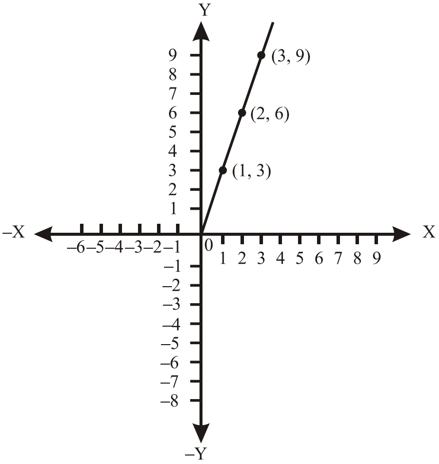
(b)

x	$y = 3 + x$
0	3
4	7
8	11



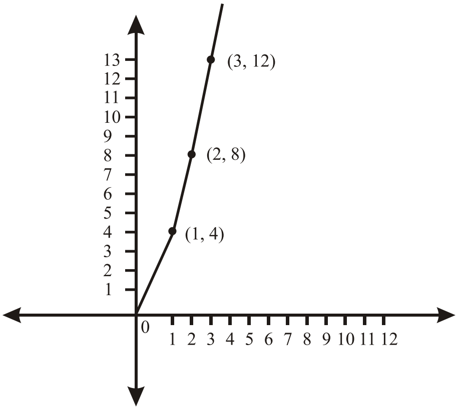
8. (a) Multiples of 3

x	1	2	3
$y = 3x$	3	6	9



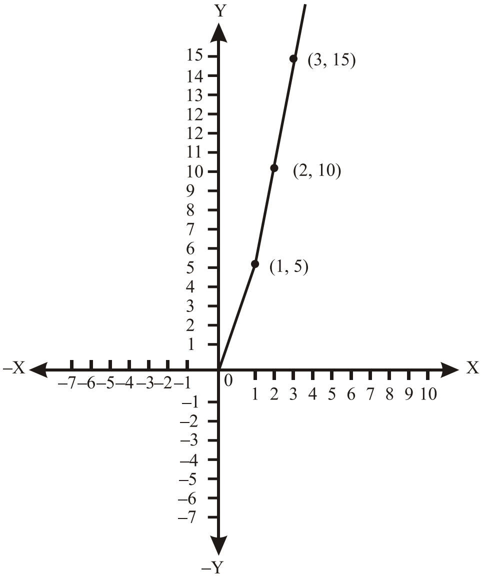
(b) Multiples of 4

x	1	2	3
$y = 4x$	4	8	12



(c) Multiples of 5

x	1	2	3
$y = 5x$	5	10	15



9. (a) I quadrant (b) III quadrant (c) II quadrant

Exercise 16.2

1. From the above graph we conclude that when

$x = 1$	$y = 2$
$x = 2$	$y = 4$
$x = -1$	$y = -2$

From here, we can draw a general relation : $y = 2x$.

Now, putting the values of x-coordinate for different point in the equation, let us find the value of y. The calculation is per formed in the table given along side :

x	y
2	6
–3	-6
5	10
-4	-8

Now, let us check whether the following points $(-3, -6)$; (5, 10) ; $(-4, -8)$, satisfy the above equation or not.

It is clear from the table that all the ordered pair except (2, 6) satisfy the equation. Therefore, all ordered pair except (2, 6) lie on the graph.

2. (a) The object moves with high speed and then slightly slowed and finally comes to rest.

(b) The object moves with non-uniform speed and comes at rest occasionally.

(c) The object starts moving after some time with high speed.

3. (a) (i) Time (minutes)

(ii) Metres above the ground.

(b) 10 minutes

(c) Speed $= \frac{1000}{10} = 100$ m/minutes

(d) 130 minutes or 2 hrs 10 minutes

(e) level flight $= (50-10)+(110-70) = (40+40)$ minutes $= 80$ minutes.

4. (i) (a) 1 hour (b) 10 : 00 am (c) 12 : 00 pm

(ii) (a) Average speed over the first hour $= 40$ km/h

(b) Average speed over the second

hour $= \frac{(160-40)}{(2-1)}$ km/h $= 120$ km/h

(c) Average speed for the last part of his journey $= \left(\frac{240-160}{5-3}\right)$ km/h

$$= \frac{80}{2} \text{ km/h} = 40 \text{ km/h}$$

MCQs

1. (b) **2.** (b) **3.** (c) **4.** (c)

17

Data Handling

Exercise 17.1

1. First we arrange the given data in ascending order :

17, 18, 22, 24, 24, 24, 26, 26, 27, 29, 30, 30, 32, 32, 36

$\therefore$ Range $= 36-17 = 19$

2. (a) The given data is discrete.

(b)

Number of wickets taken by a bowler	Tally Marks	Frequency
0	|||	3
1	||||	4
2	||||	4

3	卌 |	6
4	||	2
5	||||	4
6	||	2

3.

Number of persons	Tally Marks	Frequency
1	卌	5
2	卌 |	6
3	|||	3
4	||||	4
5	|||	3
6	|	1
7	||||	4
8	|	1
9	|||	3

(b) 4 families have members of age 4 years.
(c) 22 families have members below 7 years.
(d) 3 families have the same number of persons below 10 years.

4. First we arrange the given data in ascending order :
138, 138, 139, 140, 140, 140, 140, 146, 146, 146, 148, 148, 148, 148, 148, 148, 150, 150, 150, 150, 150, 150, 152, 152, 152, 153, 153, 153, 154, 160, 160
(a)

Height (in cm)	Tally Marks	Frequency
138	||	2
139	|	1
140	||||	4
146	|||	3
148	||||| |	6
150	||||| |	6
152	|||	3
153	|||	3
154	|	1
160	||	2
	Total	31

(b) 15 girls are of the height 150 cm and more.
(c) 7 girls are of height 145 cm and less.

Exercise 17.2

1.

Class-intervals Marks	Tally marks	Frequency
20-30	||	2
30-40	||	2
40-50	||||	4
50-60	𝍸 |	6
60-70	𝍸 ||||	9
70-80	𝍸 |||	8
80-90	||	2
90-100	||	2
	Total	35

2. (a) Class size $=$ Upper limit $-$ Lower limit $= 50-40=10$

(b) 20 is the lower limit of the class 40-50.

(c) 50 is the upper limit of the class 40-50.

(d) Class mark $=\frac{\text{upper limit } + \text{ lower limit}}{2}=\frac{20+10}{2}=\frac{30}{2}=15$

(e) 4 and 9 are the frequencies of the class-intervals 30-40 and 40-50.

3. (a) Class size $=$ upper limit $-$ lower limit $= 200-100=100$

(b) 300 is the lower limit of 300-400.

(c) 500 is the upper limit of 400-500.

(d) There are 30 doctors in the highest fee group.

(e) 90 is the frequency of the class interval 100-200.

4. First we arrange the given data in ascending order :

3, 6, 6, 8, 8, 9, 9, 11, 11, 12, 12, 13, 14, 15, 15, 15, 16, 16, 17, 17, 18, 18, 19, 21, 21, 21, 22, 22, 23, 23

Marks in test	Tally Marks	Frequency
0-5	|	1
6-10	𝍸 |	6
11-15	𝍸 ||||	9
16-20	𝍸 ||	7
21-25	𝍸 ||	7

5. First we arrange the given data in ascending order :

25, 25, 26, 27, 27, 27, 28, 28, 29, 29, 29, 29, 30, 32, 35, 35, 37, 38, 40, 40, 41, 42, 45, 47, 51, 52, 52, 53, 54, 55

Age of teachers	Tally Marks	Frequency
20-25	||	2
26-30	𝍸 𝍸 |	11

31-35	|||	3
36-40	||||	4
41-45	|||	3
46-50	|	1
51-55	~~||||~~ |	6

6. First we arrange the given data in ascending order :
68, 70, 77, 77, 82, 83, 83, 85, 85, 88, 88, 89, 89, 90, 91, 91, 92, 94, 94, 95, 96, 97, 97, 102, 103, 106, 110, 112, 112, 120

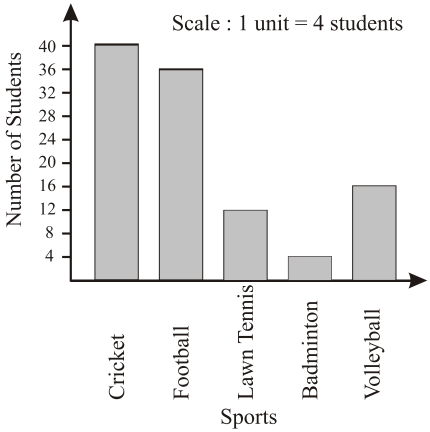
(a)

Class interval	Tally Marks	Frequency
65-70	|	1
70-75	|	1
75-80	||	2
80-85	|||	3
85-90	~~||||~~ |	6
90-95	~~||||~~ |	6
95-100	||||	4
100-105	||	2
105-110	|	1
110-115	|||	3
115-120	|	1
	Total	30

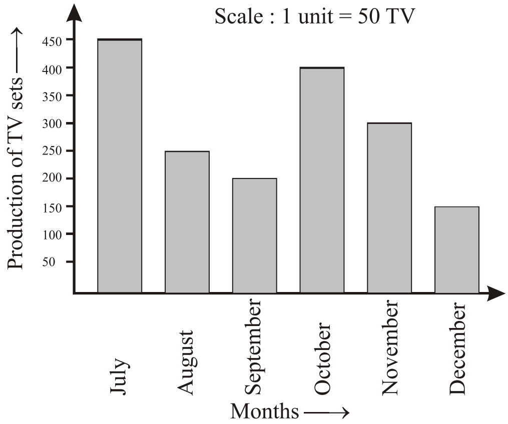
(b) 17 workers have daily income more than ₹ 89.

Exercise 17.3

1. The above information can be represented through bar graph as follows :



2. The above information can be represented through bar graph as follows :



3.

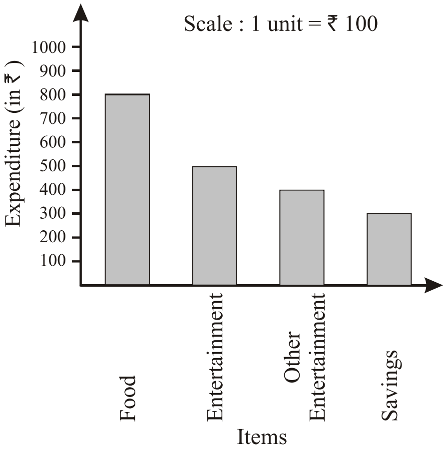
$$\text{Expenditure on food} = ₹\ 2000 \times \frac{40}{100} = ₹\ 800$$

$$\text{Expenditure on entertainment} = ₹\ 2000 \times \frac{25}{100} = ₹\ 500$$

$$\text{Expenditure on other expenditure} = ₹\ 2000 \times \frac{20}{100} = ₹\ 400$$

$$\text{Expenditure on savings} = ₹\ 2000 \times \frac{15}{100} = ₹\ 300$$

The above information can be represented through bar graph as follows :



4. (a) The bar graph represents the linking of the students for various subjects.
(b) *Y*-axis represents number of students.
(c) 75 students like Physics subject.
(d) Maths is the favourite subject of most of the students.
(e) 20 more students like Maths in comparison to Biology.

5. (a) In 2005, the pass percentage of class *X* is more as comparison to class XII.
(b) In 2008, the pass percentage of class XII is maximum.
(c) In 2005, $\text{Difference} = (90-80)\% = 10\%$

In 2006, Difference $= (90 - 75)\% = 15\%$

In 2007, Difference $= (90 - 85)\% = 5\%$

In 2008, Difference $= (95 - 80)\% = 5\%$

So, in 2006, the difference is maximum.

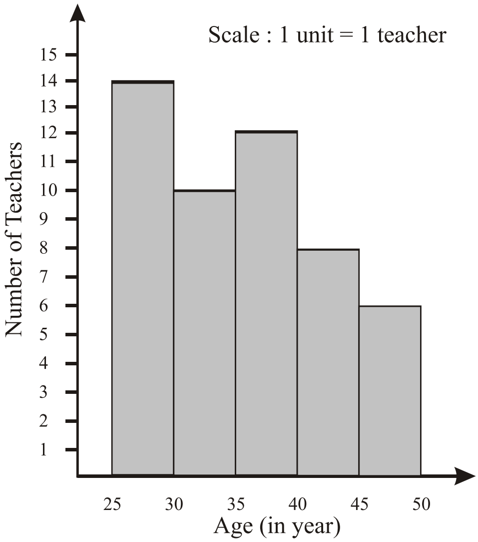
(d) Average passing percentage for class X $= \dfrac{90 + 75 + 85 + 90}{4} = 85\%$

Average passing percentage for class XII $= \dfrac{80 + 90 + 90 + 95}{4} = 88.75\%$

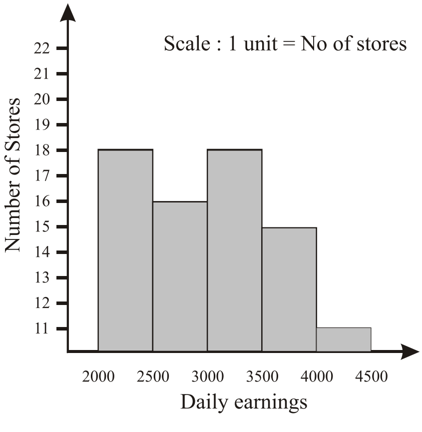
So, class XII has more passing percentage.

Exercise 17.4

1. The above information can be represented through histogram as follows :



2. The above information can be represented through histogram as follows :

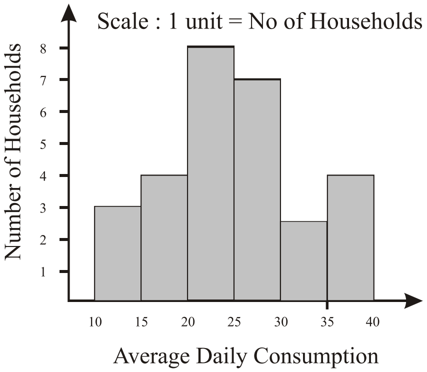


3.

Average Daily Consumption of wheat flour	Tally Marks	Frequency
10-15	|||	3

15-20	||||	4
20-25	|||| |||	8
25-30	|||| ||	7
30-35	|||	3
35-40	~~||||~~	5

The above information can be represented through his to gram as follows :



4. (a) The data represents the sales of coats of different size.
(b) (44-46) class was the least number of coats sold.
(c) (40-42) class was the least number of coats sold.
(d) In sizes (40-42) were sold 300 coats.
(e) 900 coats of sizes between 34 and 44 were sold.

Exercise 17.5

1. Degree of $A = 35\% \times 360° = \frac{35}{100} \times 360° = 126°$

Degree of $B = 20\% \times 360° = \frac{20}{100} \times 360° = 72°$

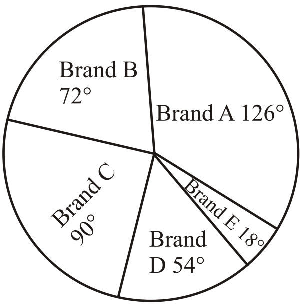
Degree of $C = 25\% \times 360° = \frac{25}{100} \times 360° = 90°$

Degree of $D = 15\% \times 360° = \frac{15}{100} \times 360° = 54°$

Degree of $E = 5\% \times 360° = \frac{5}{100} \times 360° = 18°$

The table and pie chart are :

Brand of Soap	% of consumption	Degrees on Pie Chart
A	35	126°
B	20	62°
C	25	90°
D	15	54°
E	5	18°



2. Let the total number of persons be x.

Then, 30% of $x = 600$ (Given)

$$\frac{30}{100} \times x = 600$$

$$x = 20 \times 100 = 2000$$

$$\text{No. of Engineers} = 20\% \text{ of total no. of persons} = \frac{20}{100} \times 2000 = 400$$

$$\text{No. of Architects} = 10\% \text{ of } x = \frac{10}{100} \times 2000 = 200$$

$$\text{No. of Doctors} = 15\% \text{ of } x = \frac{15}{100} \times 2000 = 300$$

$$\text{No. of Lawyears} = 25\% \text{ of } x = \frac{25}{100} \times 2000 = 500$$

3. Convert the numerical value into percentage.

Transport to school	No. of students	Percentage
On foot	650	$\frac{650}{2100} \times 100\% = 31\%$
Bus	1200	$\frac{1200}{2100} \times 100\% = 57\%$
Bicycle	100	$\frac{100}{2100} \times 100\% = 5\%$
Private van	100	$\frac{100}{2100} \times 100\% = 5\%$
Bike	50	$\frac{50}{2100} \times 100\% = 2\%$
	Total	

Now, convert the percentage into degrees :

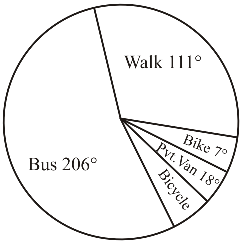
Since, a circle represents the total number of students, *i.e.*, 100% and the total number of degrees in a circle is 360°.

Therefore, 100% is represented by 360°.

So, 1% is represented by $\frac{360°}{100}$ *i.e.*, 3.6°.

Transport to school	No. of students	Percentage	Degrees	Round off
On foot	650	31%	$31 \times 3.6 = 111.6°$	111°
Bus	1200	57%	$57 \times 3.6 = 205.2°$	206°
Bicycle	100	5%	$5 \times 3.6 = 18°$	18°
Private van	100	5%	$5 \times 3.6 = 18°$	18°
Bike	50	2%	$2 \times 3.6 = 7.2°$	7°
Total	2100			360°

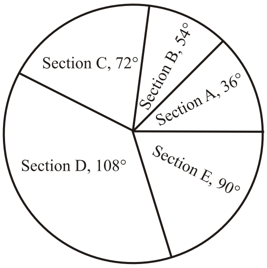
So, the pie chart are :



Similarly,

Sections	No. of students passed with distinction	%	Degree	Round off
A	10	$\frac{10}{100} \times 100\% = 10\%$	$10 \times 3.6° = 36°$	36°
B	15	$\frac{15}{100} \times 100\% = 15\%$	$15 \times 3.6° = 54°$	54°
C	20	$\frac{20}{100} \times 100\% = 20\%$	$20 \times 3.6° = 72°$	72°
D	30	$\frac{30}{100} \times 100\% = 10\%$	$30 \times 3.6° = 108°$	108°
E	25	$\frac{25}{100} \times 100\% = 25\%$	$25 \times 3.6° = 90°$	90°
Total	100			360°

So, The pie chart are :



4. The degree of chemistry $= 360° - (100° + 75° + 100°) = 360° - 275° = 85°$

Let the total marks of all subject be x.

Then,
$$\frac{80}{x} \times 360° = 100°$$
$$\frac{80 \times 360°}{x} = 100°$$
$$x = 288°$$
$$\text{The marks in chemistry} = \frac{85°}{360°} \times 288 = \frac{24480°}{360°} = 68$$

5.
$$\text{Percentage of poetry books} = \frac{\text{No. of poetry books}}{\text{Total books}}$$
$$= \left(\frac{180}{400} \times 100\right)\% = 45\%$$

$\therefore$ the degree of poetry books $= 45 \times 3.6 = 162°$

Percentage of adventure books $= \left(\frac{60}{400} \times 100\right)\% = 15°\,\%$

$\therefore$ the degree of adventure books $= 15 \times 3.6 = 54°$

Percentage of science books $= \left(\frac{80}{400} \times 100\right)\% = 40\%$

$\therefore$ the degree of science books $= 40 \times 3.6 = 144°$

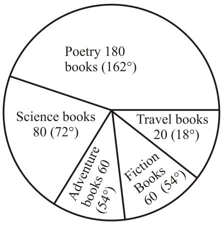
Percentage of travel books $= \left(\frac{20}{400} \times 100\right)\% = 5\%$

$\therefore$ the degree of Travel books $= 5 \times 3.6° = 18°$

The percentage of Fiction books $= \left(\frac{60}{400} \times 100\right)\% = 15\%$

$\therefore$ the degree of Fiction books $= 15 \times 3.6° = 54°$

The pie chart are :



MCQs

1. (b) **2.** (c) **3.** (d) **4.** (b) **5.** (d)

18

Probability

Exercise 18.1

1. (a) Next year, the sun will set in the south.

Because sun will never set in the south.

$\therefore$ $P(E) = 0$

(b) Your teacher will teach you maths for 200 years.

Because average life of a human body is 80 year and nobody can be alive till 200 years.

$\therefore$ $P(E) = 0$

(c) You get a head when you toss a coin.

Because there is only two event possible here, either you get head or tail.

So, $P(E) = \frac{1}{2}$

2. (a) I

No. of favourable outcomes -1

Total no. of possible outcomes $= 6$

$\therefore$ $P(I) = \frac{1}{6}$

(b) Grey colour

No. of favourable outcomes $= 2$

Total no. of outcomes $= 6$

So, P (grey colour) $= \frac{2}{6} = \frac{1}{3}$

(c) Pink colour

No. of favourable outcomes $= 4$

Total No. of outcomes $= 6$

So, P (pink colour) $= \frac{4}{6} = \frac{2}{3}$

(d) Number less than IV.

No. of favourable outcomes $= 3$

Total no. of outcomes $= 6$

So, $P\,(IV) = \frac{3}{6} = \frac{1}{2}$

(e) A number less than VI

No. of favourable outcomes $= 5$

Total no. of outcomes $= 6$

So, $P\,(VI) = \frac{5}{6}$

3. (a) Prime No.

A die as 6 faces and the no is 1, 2, 3, 4, 5, 6 are on face each.

No. of favourable outcomes $= 3$

Total no. of outcomes $= 6$

$\therefore$ $P\,(E) = \frac{3}{6} = \frac{1}{2}$

(b) 2 or 4

No. of favourable outcomes $= 2$

Total no. of outcomes $= 6$

$\therefore$ $P\,(E) = \frac{2}{6} = \frac{1}{3}$

(c) A multiple of 2 or 3.

No. of favourable outcomes $= 4$

Total no. of outcomes $= 6$

$\therefore$ $P\,(E) = \frac{4}{3} = \frac{2}{3}$

4. Square no. are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

No. of favourable outcomes $= 10$

Total outcomes $= 100$

$P\,(E) = \frac{10}{100} = \frac{1}{10}$.

5.

Numbers	1	2	3	4	5	6	7	8	9	10
Frequency	1	4	5	0	2	5	2	1	3	2

We know that,

$$\text{Probability of any event} = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$$

So, $P(1) = \frac{1}{25}$ $\quad P(2) = \frac{4}{25}$ $\quad P(3) = \frac{5}{25} = \frac{1}{5}$

$P(4) = \frac{0}{25} = 0$ $\quad P(5) = \frac{2}{25} = \frac{2}{25}$ $\quad P(6) = \frac{5}{25} = \frac{1}{5}$

$P(7) = \frac{2}{25}$ $\quad P(8) = \frac{1}{25}$ $\quad P(9) = \frac{3}{25}$

$P(10) = \frac{2}{25}$

6. (a) There are 26 black cards in a pack of 52 cards.

No. of favourable outcomes $= 26$

Total no. of outcomes $= 52$

So, $P(\text{black}) = \frac{26}{52} = \frac{1}{2}$

(b) There are 13 spades in a pack of 52 cards.

No. of favourable outcomes $= 13$

Total no. of outcomes $= 52$

So, $P(\text{spade}) = \frac{13}{52} = \frac{1}{4}$

(c) There are 4 kings in a pack of 52 cards.

No. of favourable outcomes $= 4$

Total no. of outcomes $= 52$

$P(\text{king}) = \frac{4}{52} = \frac{1}{13}$

(d) There are two red jack in a pack of 52 cards.

No. of favourable outcomes $= 2$

Total no. of outcomes $= 52$

$P(\text{Red Jack}) = \frac{2}{52} = \frac{1}{26}$

7. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Multiple of 4 $= 4, 8, 12$

No. of favourable outcomes $= 3$

Total no. of outcomes $= 15$

So, $P(E) = \frac{3}{15} = \frac{1}{5}$

8. Do it yourself.

MCQs

1. (a) **2.** (d) **3.** (c) **4.** (b) **5.** (c)